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# Conference Paper

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C-BAND RADAR-BEACON TRACKING FOR PROJECT MERCURY

## CASE FILE COPY

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## ABSTRACT

As advisors to the National Aeronautics and Space Administration, Lincoln Laboratory performed an analysis of the Mercury beacon-tracking capabilities. Calculations of signal strength to be expected for orbital flights were made. As a result of the marginal signal levels which were computed, a program of improvements was initiated.

Sub-orbital flight-test data were reduced and compared with computed values. Also extrapolations were made to the orbital cases. As a result of the test data, specific recommendations regarding antenna coverage were made. Orbital flights with and without recommended antenna modifications were performed, which afforded a unique "before" and "after" comparison.

## C-BAND RADAR-BEACON TRACKING FOR PROJECT MERCURY

### I. INTRODUCTION

In making recommendations to the National Aeronautics and Space Administration (NASA) regarding the Mercury Ground Tracking Range, Lincoln Laboratory initiated a signal-level analyses of both C- and S-band radar-to-beacon,<sup>1,2</sup> beacon-to-radar links.

It became evident at that early stage that a number of difficulties existed. Our attention focused on C-band tracking problems because the C-band monopulse tracking radar (AN/FPS-16) used in the Mercury program is inherently more accurate than its S-band conically-scanned counterpart, VERLORT. It was felt that since time and resources were limited, efforts could best be utilized by concentrating on the C-band tracking.\*

It is the purpose of this paper to indicate:

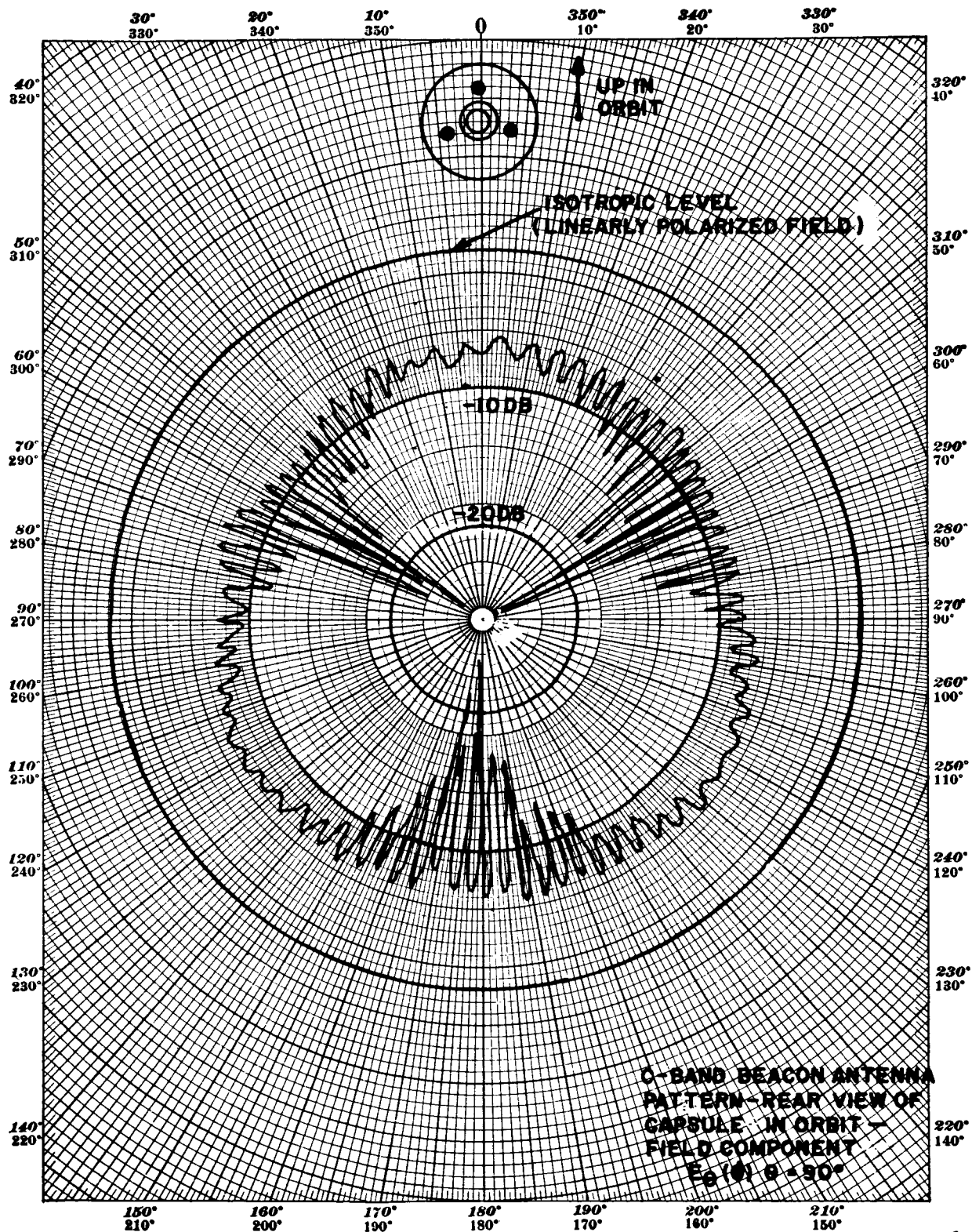
- a) Some of the problems involved
- b) Interpretation of observed tests
- c) Recommended remedial action
- d) Test results of sub-orbital flights, and orbital flights with and without recommended modifications.

### II. CAPSULE BEACON ANTENNA COVERAGE

Figure 1 is a plot of C-band antenna coverage in a plane perpendicular to the longitudinal axis of the capsule. The relative position of the three circularly-polarized helical cavities, which comprise the antenna system, are

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\*Some effort was expended in developing an S-band beacon with a relatively high triggering sensitivity. See Reference No. 2, pp. 10-13.



C42-216

FIGURE 1

also designated in the same figure.

It can be seen that severe interference regions exist every  $120^\circ$ , midway between helical cavities. Possible deleterious effects of such an antenna arrangement are twofold:

- a) The nulls of the interference regions may be deep enough to cause erratic triggering of the beacon. This situation is especially serious in cases where the angular line-of-sight to the capsule is constant for long periods of time.
- b) If such an antenna system has angular motion with respect to the ground tracking radar, the interference regions may effectively scan the tracking radar. If the tracking radar is a monopulse radar (as is the AN/FPS-16 in the C-band Mercury system), it is possible for a serious "glint" problem to arise. In this instance, the phase reversal present from one side of a null to the other provides an effective tilt in the phase front emanating from the capsule. Thus, it is possible for a monopulse radar to have serious<sup>3</sup> angular perturbations.

Figure 2 illustrates a more fundamental failing. The ablative heat shield at the blunt end of the capsule effectively blocks the coverage in that general direction. Since the capsule travels blunt end first in orbit, it is apparent that effective radar acquisition of the capsule is compromised.

### III. SYSTEM SIGNAL LEVEL ANALYSIS

Early in the Mercury effort, signal level analyses were made for the radar-beacon, beacon-radar tracking links. These early efforts had assumed a nominal value for beacon-antenna gain. After the receipt of complete antenna patterns, it became possible to consider the geometry of the capsule with regard to the

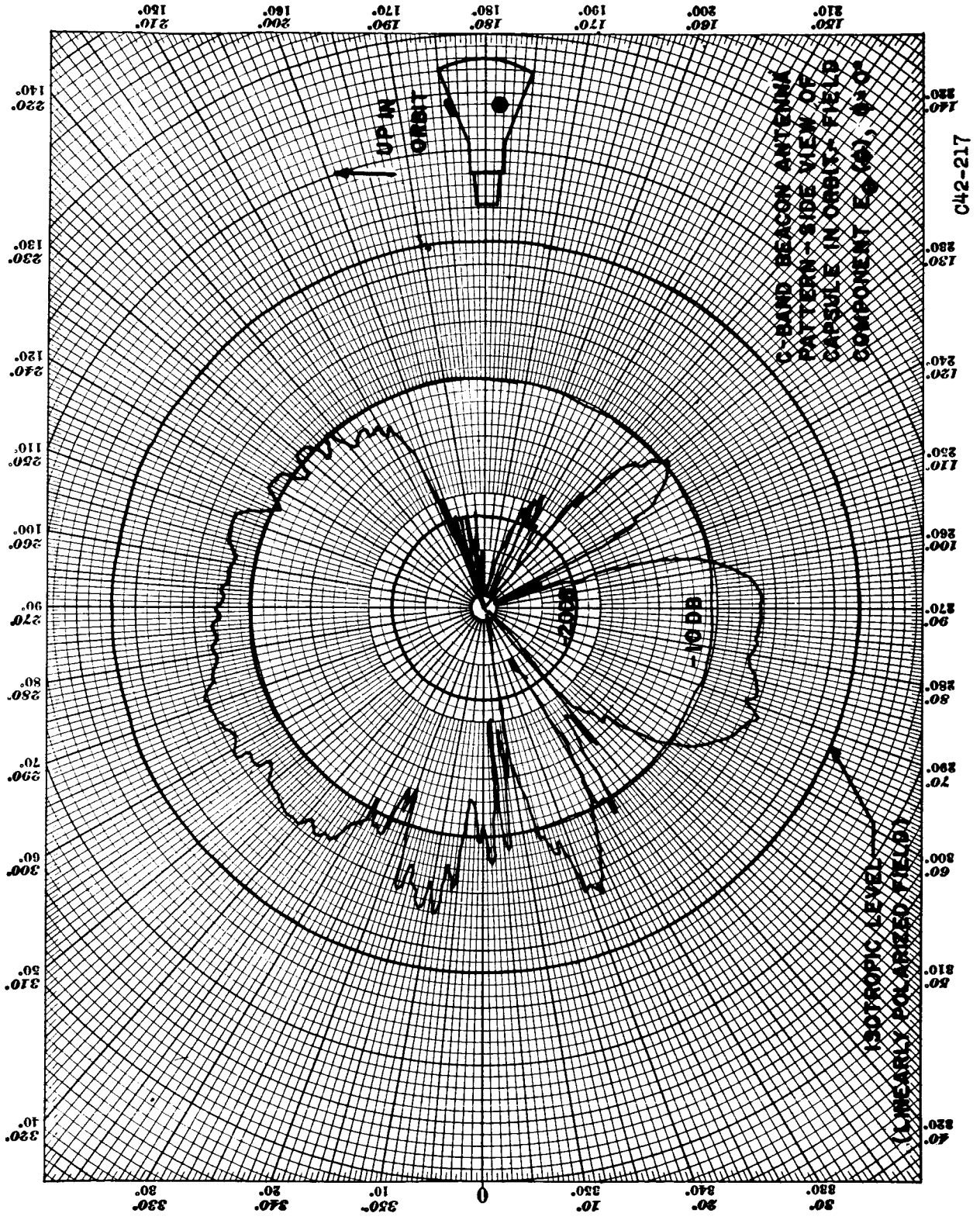


FIGURE 2

tracking radar. The parameters of the Mercury beacon-radar links are listed in Table I and Table II.

TABLE I  
RADAR-BEACON LINK

|                                |   |
|--------------------------------|---|
| Radar peak power out           | +84 dbm (250 Kw)                                  |
| Radar RF loss                  | -2 db   |
| Radar antenna gain             | +44 db/(linearly-polarized<br>lossless isotrope)  |
| Polarization loss              | -3 db   |
| RF loss in capsule             | -2.5 db   |
| Space loss                     | -[112.5 + 20 log R (nm)] db                       |
| Beacon antenna gain            | <u>from antenna patterns</u>                      |
| TOTAL POWER AT BEACON RECEIVER | +120.5 + (space loss + beacon<br>antenna gain) db |

TABLE II  
BEACON-RADAR LINK

|                               |  |
|-------------------------------|--|
| Beacon peak power out         | +56 dbm (375 watts)                              |
| Beacon RF loss                | -2.5 db  |
| Radar antenna gain            | +44 db/(linearly-polarized<br>lossless isotrope) |
| Polarization loss             | -3 db  |
| Radar RF loss                 | -2 db  |
| Space loss                    | -[112.5 + 20 log R (nm)] db                      |
| Beacon antenna gain           | <u>from antenna patterns</u>                     |
| TOTAL POWER AT RADAR RECEIVER | +92.5 + (space loss + beacon<br>antenna gain) db |

Using the parameters established in Tables I and II and taking into account the geometry of the launch and orbital phases of flight, a number of plots of signal-to-noise versus time relationships were derived.<sup>4</sup>

Figure 3 describes a computed overhead pass at Point Arguello, California. The aperture blocking due to the capsule heat shield is such that acquisition by the AN/FPS-16 radar would not have been possible until the capsule had approached to within approximately 250 nm, due to an inadequate interrogating signal at the capsule beacon. Note that this plot assumes the originally conceived  $14^\circ$  angle of attack. It was due to the above and other considerations that the angle of attack for the manned flights was changed from  $14^\circ$  to  $34^\circ$ .<sup>5</sup>

#### IV. THE ABORT DECISION AT BERMUDA

The tracking AN/FPS-16 radar at Bermuda, together with an IBM 7090 computer, is charged with the responsibility of determining if the predicted orbit (as computed at Bermuda) will be adequate for the mission. This important decision must be made within approximately one-half minute after the time the capsule is inserted into orbit.

The curve plotted in Fig. 4 describes the computed signal-to-noise ratios at Bermuda for the capsule during this interval. During this important phase of the flight, the capsule assumes a "ready" retrofire attitude of  $34^\circ$ , by rotating  $180^\circ$  so that the blunt end of the capsule faces toward Bermuda.

The dissimilarity of the plots of signal-to-noise ratios for the AN/FPS-16 receiver signals (IF bandwidths of 2 Mcps and 8 Mcps) and the beacon signal threshold margin in these figures is distinct. The basis for this difference is due to the great difference in antenna patterns. The antenna system, which is relatively frequency sensitive, was tuned for an optimum at the beacon transmitter frequency. Thus the beacon receiver patterns are considerably poorer than its transmitter patterns. The beacon magnetron frequency drift specifications call for a frequency tolerance of  $\pm 4$  Mcps for the Avion 149C beacon.

POINT ARGUELLO 34.58°N, 120.58°W, REV. #3  
14° ANGLE OF ATTACK

C-BAND NO MARGINS ASSUMED

COMPUTED RADAR-BEACON, BEACON-RADAR  
S/N VS TIME

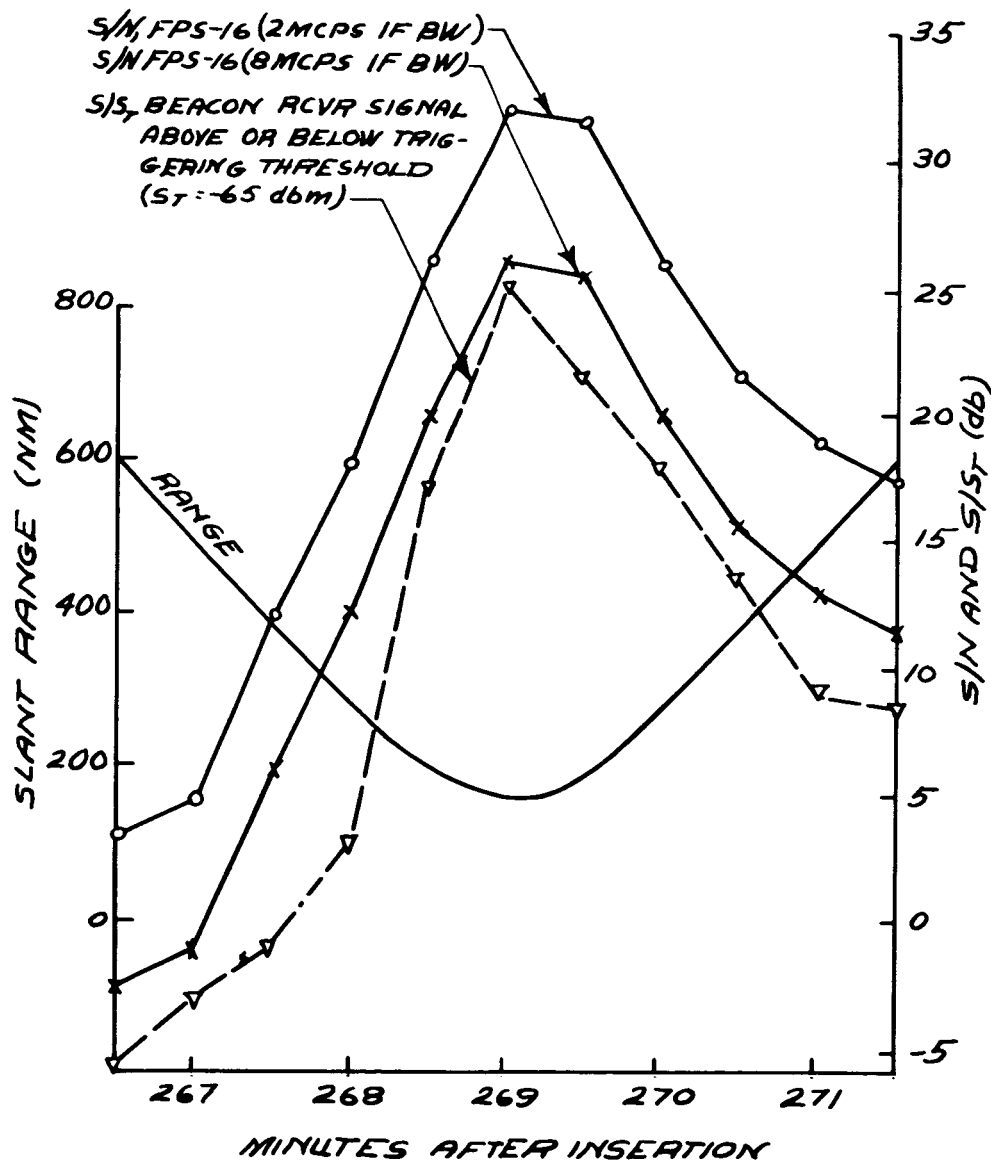


FIGURE 3

C42-337

BERMUDA 32.33°N, 64.70°W, REV. NO. 1

COMBINED C.C.W TURN AND TILT TO 34°

RADAR-BEACON, BEACON-RADAR LINK

COMPUTED S/N VS TIME

C-BAND, FPS-16

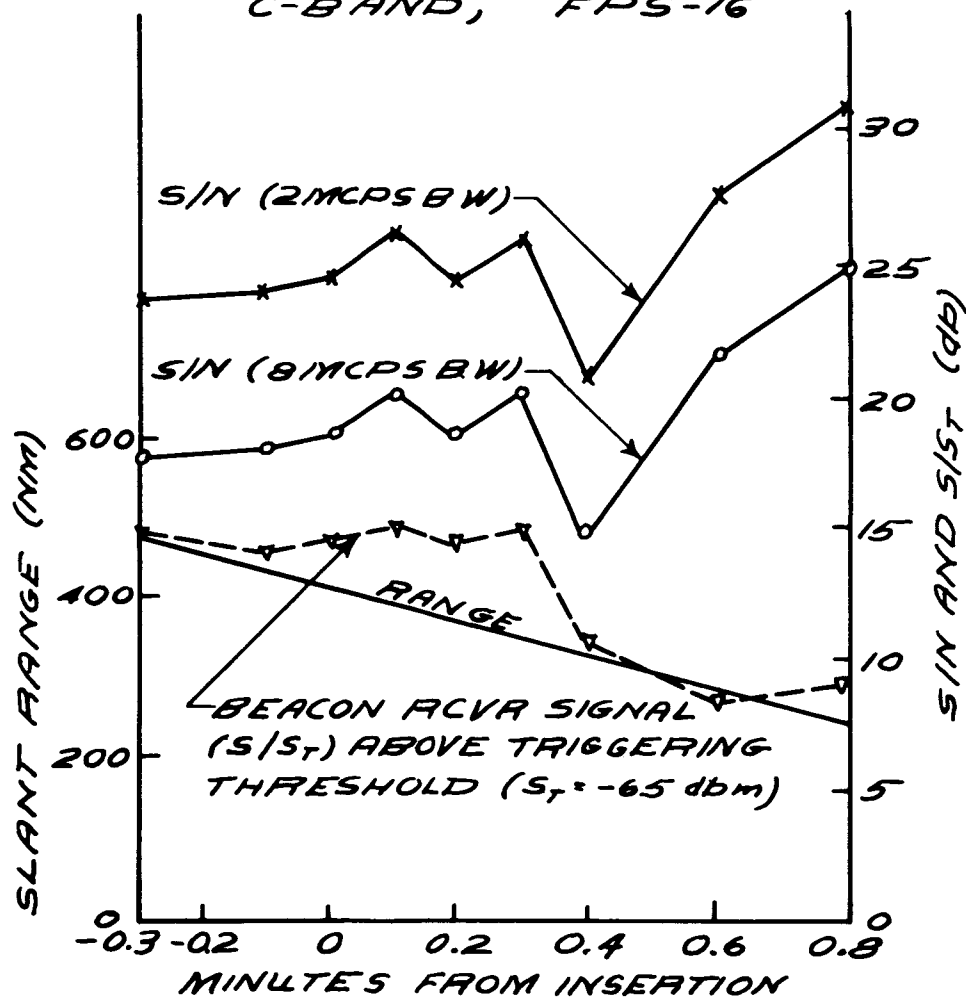


FIGURE 4

C42-338

This possible total frequency drift of 8 Mcps could result in a phase shift between elements of approximately  $30^\circ$  for an antenna-element circumferential separation of roughly 180 cm. It would appear, therefore, that variations in the beacon transmitter patterns may be expected as its magnetron warms up and changes frequency. Also, changes in the AN/FPS-16 transmitter frequency will affect the beacon receiver patterns.

There are several parameters at capsule insertion into orbit that define its flight path. They are as follows:

- a) Height
- b) Latitude
- c) Longitude
- d) Direction angle
- e) Velocity
- f)  $\gamma$  (angle between flight path and the plane formed by the local horizontal at that point)

These parameters are effectively controlled by the GE-Burroughs guidance system of the Atlas. It remains for the tracking system at Bermuda to firmly establish the orbit by measuring the velocity and  $\gamma$  angle after capsule separation. The radar at Bermuda is a range and angle (azimuth and elevation) measuring device. In computing the velocity and  $\gamma$  angle, parameters (a), (b), (c), and (d) will also be computed at Bermuda. Thus a check of all the insertion parameters is automatically performed.

The order of accuracy required for velocity and  $\gamma$  angle are assumed to be:

|                |                          |
|----------------|--------------------------|
| Velocity       | $\pm 5 \text{ ft/sec}^6$ |
| $\gamma$ angle | $\pm 0.01^\circ$         |

Table III enumerates various combinations of RMS angular noise (milliradians), duration of sample time (sec), and sampling rate (samples per sec), which result in corresponding accuracies of computed velocities and  $\gamma$  angles.

TABLE III

COMBINATIONS OF RADAR RMS ERRORS AND RESULTING COMPUTED ACCURACIES IN VELOCITIES AND  $\gamma$  ANGLES (RMS RANGE NOISE ASSUMED CONSTANT AT 45 FEET)

| RMS Angular Noise<br>(milliradians) | Sampling<br>Time<br>(sec) | Sampling Rate<br>(samples/sec) | Velocity Accuracy<br>(feet/sec) | $\gamma$<br>(degrees) |
|-------------------------------------|---------------------------|--------------------------------|---------------------------------|-----------------------|
| 0.1                                 | 30                        | 10                             | 0.7                             | 0.004                 |
| 0.3                                 | 30                        | 10                             | 1.5                             | 0.012                 |
| 1.0                                 | 30                        | 10                             | 5.5                             | 0.034                 |
| 0.1                                 | 20                        | 10                             | 1.2                             | 0.006                 |
| 0.3                                 | 20                        | 10                             | 2.8                             | 0.020                 |
| 1.0                                 | 20                        | 10                             | 9.3                             | 0.066                 |
| 0.1                                 | 30                        | 3                              | 1.1                             | 0.006                 |
| 0.3                                 | 30                        | 3                              | 3.2                             | 0.021                 |
| 1.0                                 | 30                        | 3                              | 6.1                             | 0.053                 |
| 0.1                                 | 20                        | 3                              | 1.9                             | 0.012                 |
| 0.3                                 | 20                        | 3                              | 5.4                             | 0.034                 |
| 1.0                                 | 20                        | 3                              | 17.7                            | 0.127                 |

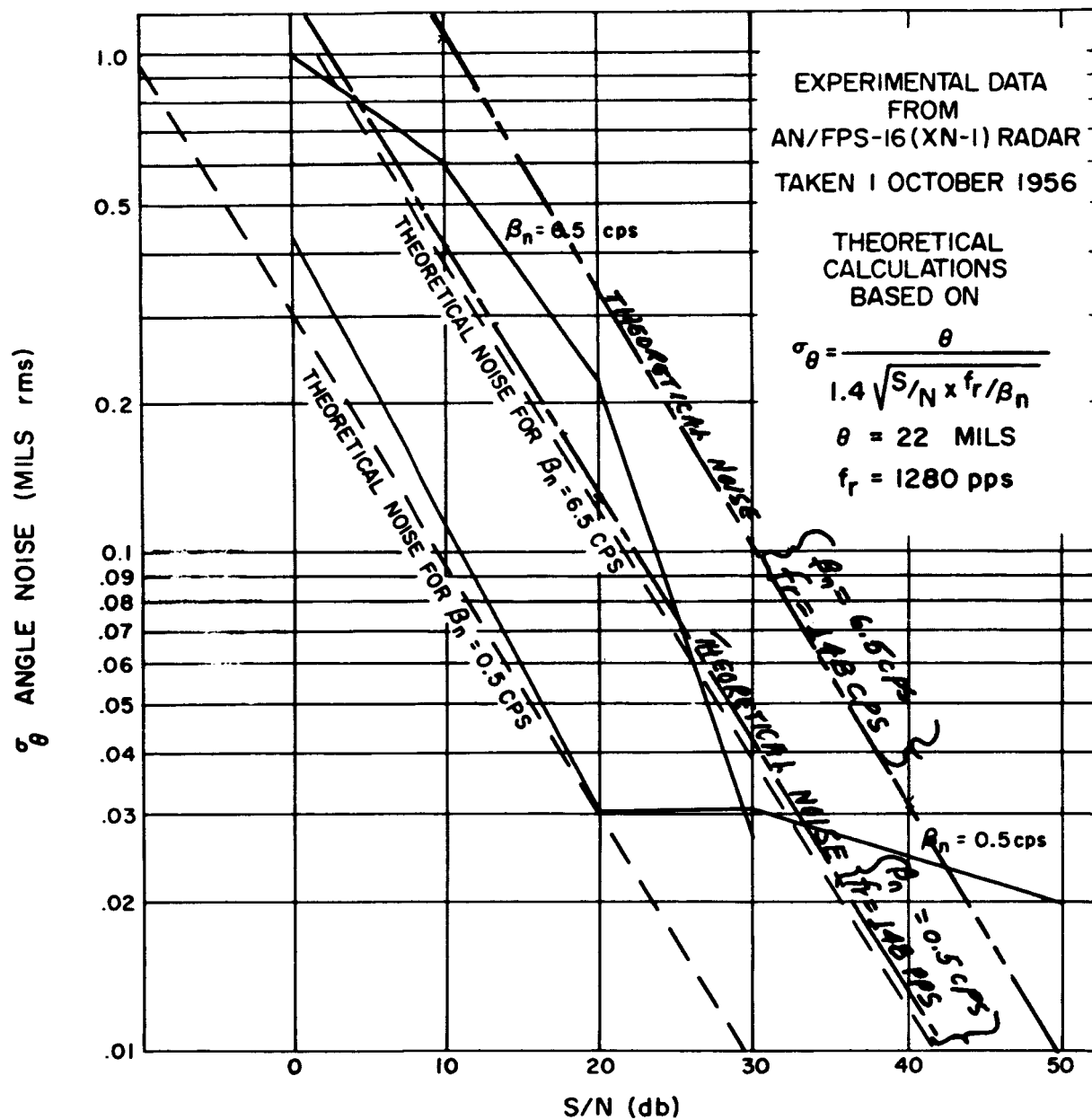
Three combinations of RMS angular noise, sampling time, and sampling rate of radar data that enable one to compute the velocity and  $\gamma$  angle within the specified accuracy requirements are shown in Table III (boxed). In all three instances, one requires radar data with no more than 0.1 milliradian RMS angular

noise for periods of no less than twenty seconds. Translated to Figs. 5 and 6\*, signal-to-noise ratios for 0.1 milliradian (.098 mils)\*\* RMS noise in azimuth correspond to 22 db (servo bandwidth = 0.5 cps) and 30 db (servo bandwidth = 6.5 cps). For RMS angular noise in elevation, the corresponding figures are 21 db (servo bandwidth = 0.7 cps) and 29 db (servo bandwidth = 5 cps). Figure 7\* shows that a signal-to-noise ratio of 21 db provides less than 2 yards RMS range noise, substantially less than the 45 feet assumed in Table II. For the AN/FPS-16, it can be seen that a minimum of 21 db is required for at least twenty seconds. Since Fig. 5 indicates a theoretical margin of only 3 db for the 2 Mcps IF and -3 db for the 8 Mcps IF, it was established that the abort decision at Bermuda would be based on marginal data. It remained for the actual test results to validate our computations and apprehensions.

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\*Figures 5, 6, and 7 are taken from "Final Report Instrumentation Radar AN/FPS-16 (XN-1) Evaluation and Analysis of Radar Performance," Radio Corporation of America, Moorestown, New Jersey, Figures 4-30, 4-31, and 6-1, respectively.

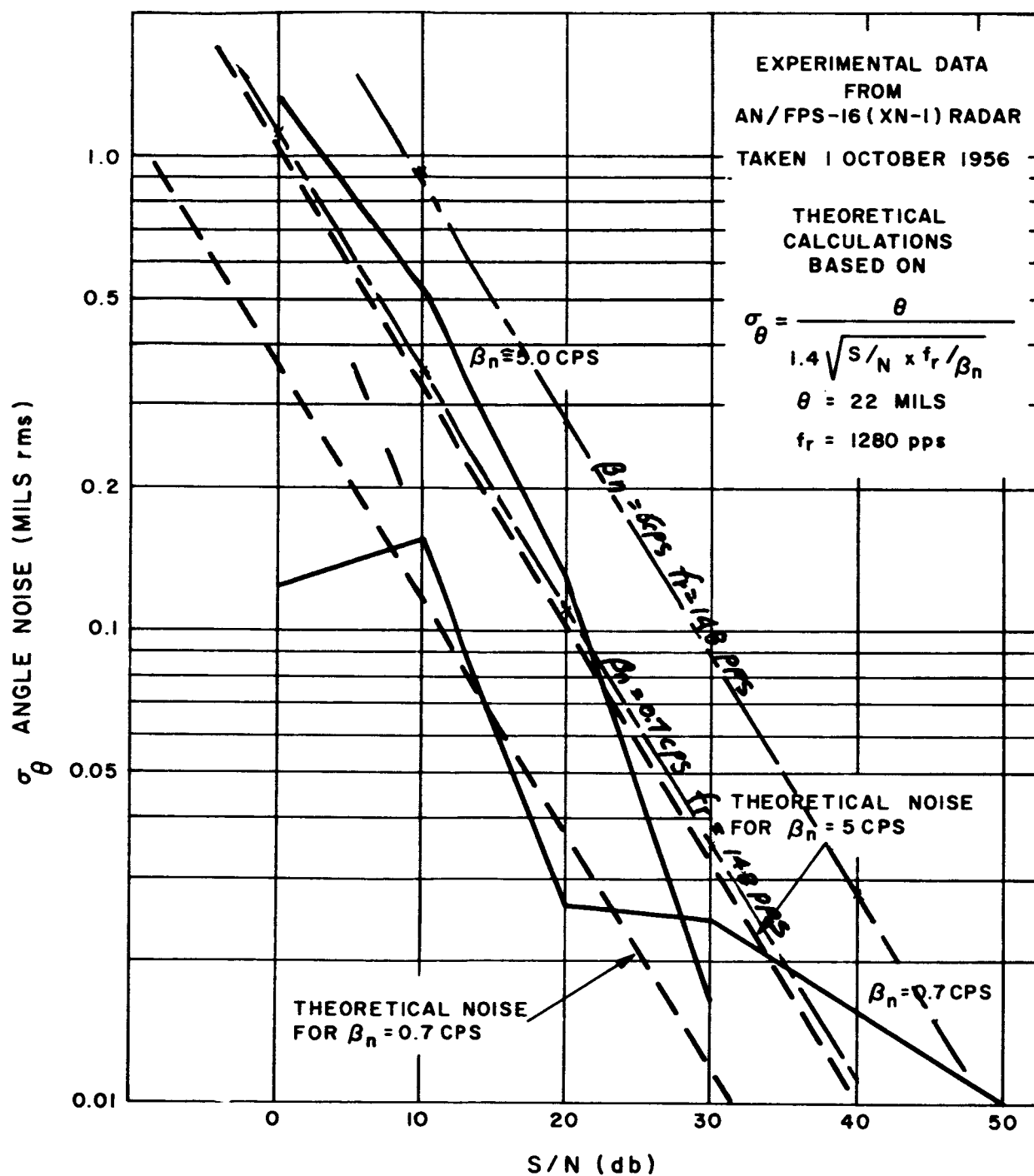
\*\*1 mil = 56.3/57.3 milliradians.



Azimuth Angle Noise vs RF  
Signal-to-Noise Ratio

C42-223

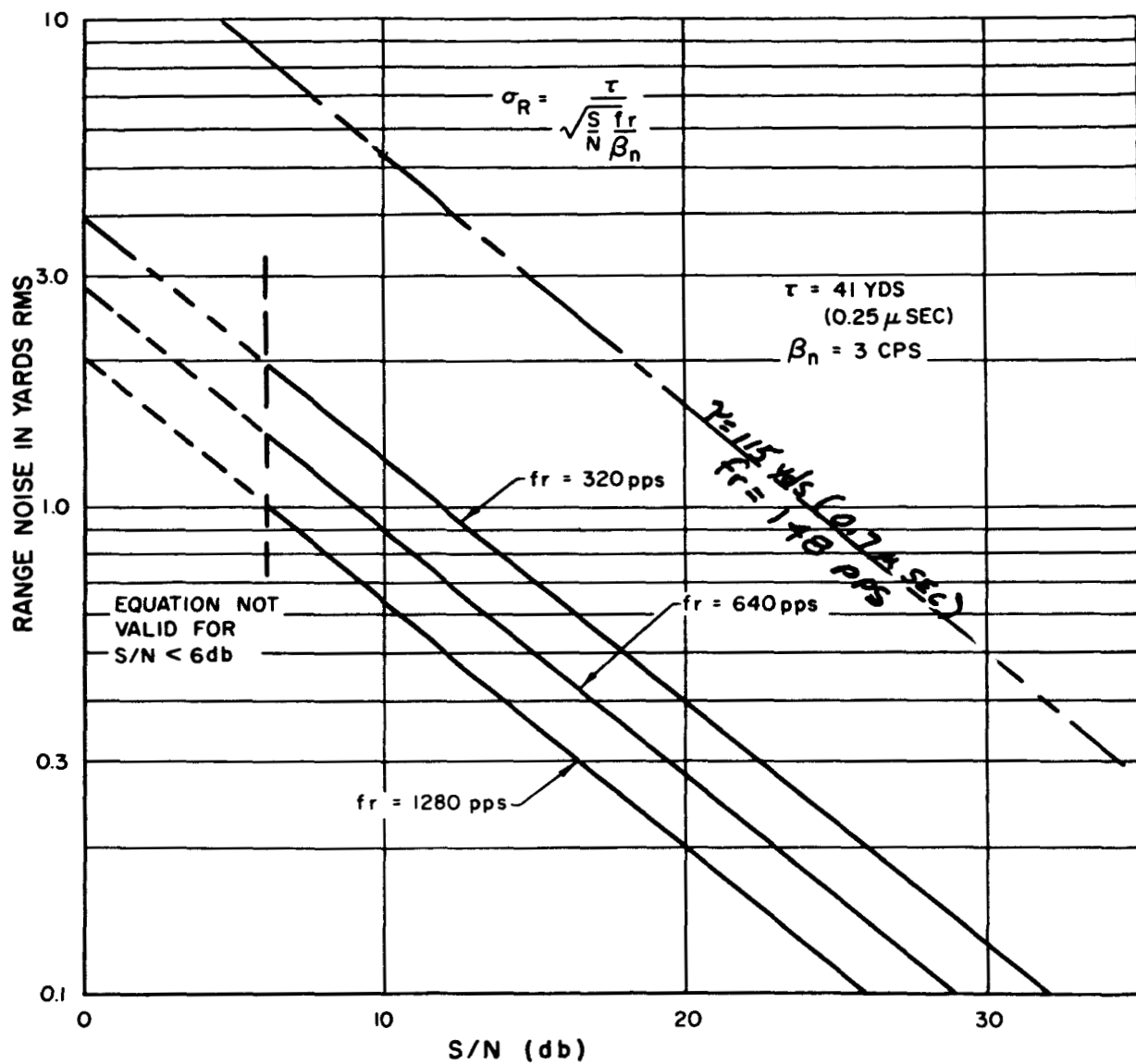
FIGURE 5



Elevation Angle Noise vs RF  
Signal-to-Noise Ratio

C42-224

FIGURE 6



Range Tracking Noise vs Signal Strength

C42-225

FIGURE 7

## V. MERCURY SUB-ORBITAL FLIGHTS

The Mercury program for sub-orbital flights commenced December 19, 1960, when a Mercury-Redstone missile was launched from Cape Canaveral (MR-1A).<sup>7</sup> The flight was programmed to be relatively short with impact planned to be about 200 nm down range. This flight was particularly significant to the Mercury Ground Range for two reasons:

- a) The flight validated the techniques and assumptions employed in the calculations of signal-to-noise ratios for the Mercury capsule in orbit based on the comparison of predicted and actual values obtained from the MR-1A test.
- b) The flight emphasized inadequacies in beacon tracking.

Figure 8 is a record of signal-to-noise ratios computed and actually obtained at Cape Canaveral. The correlation of the experimental and predicted values are self evident. The effects of capsule antenna lobes are also quite evident. It is interesting to note the effects of antenna lobes in the signal-to-noise recordings.

Figure 9 is very informative since it demonstrates the effects of antenna nulls in both the radar-beacon link and in the beacon-radar link. The relatively slowly varying signal level is due to antenna lobes. These fluctuations in this portion of the flight require about one to three seconds per period. There are also, however, superimposed on the slower variations relatively rapid losses of signal which transpire in about 1/4 second.

These rapid fluctuations are due simply to a lack of adequate triggering signal at the capsule beacon. Since the signal required for triggering is not achieved in the nulls of the antenna pattern and since these are rather sharp, the time that the beacon does not respond is small. The fact that the beacon fails to reply and time of occurrence of nulls in the beacon-transmit pattern

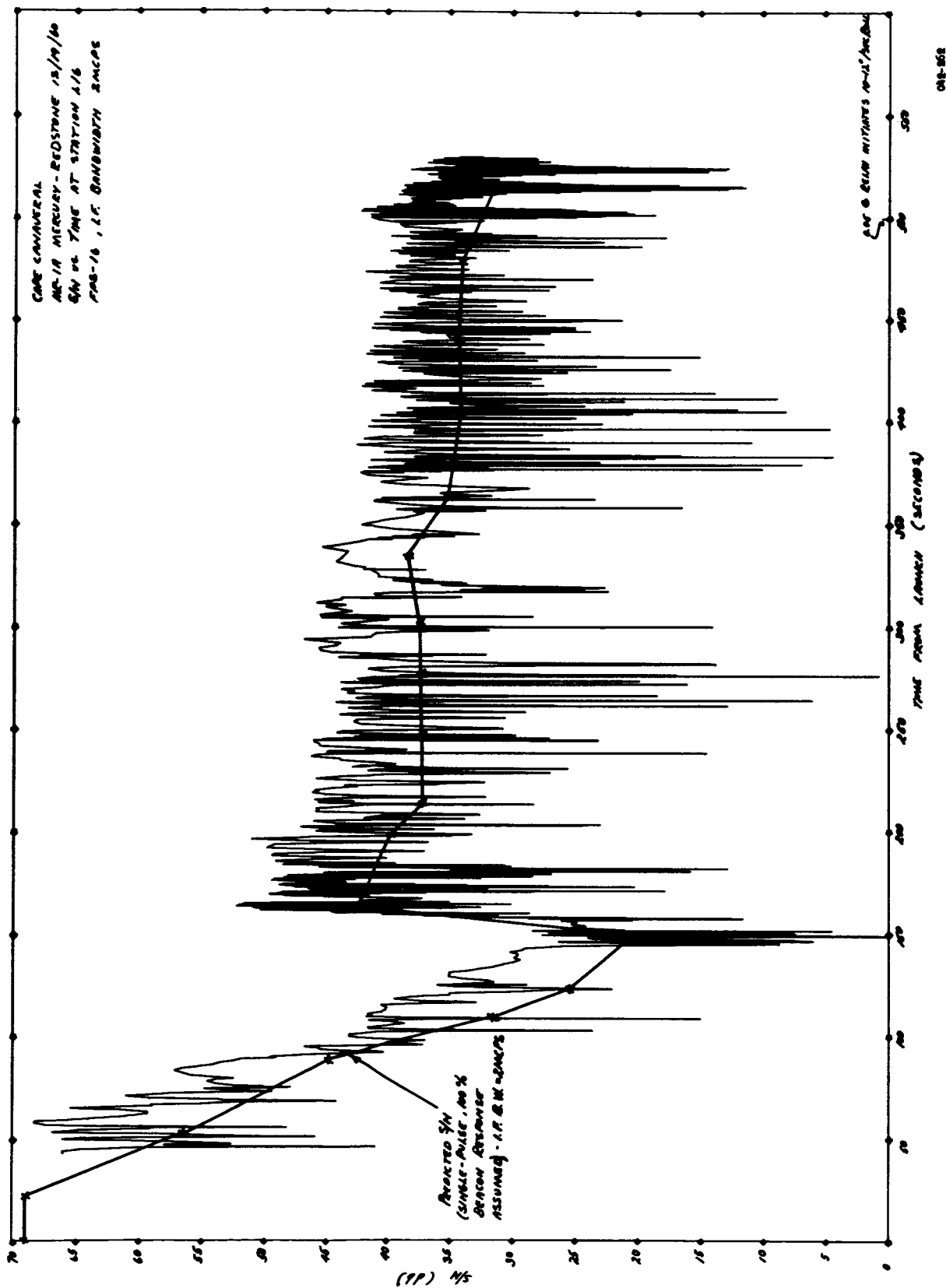
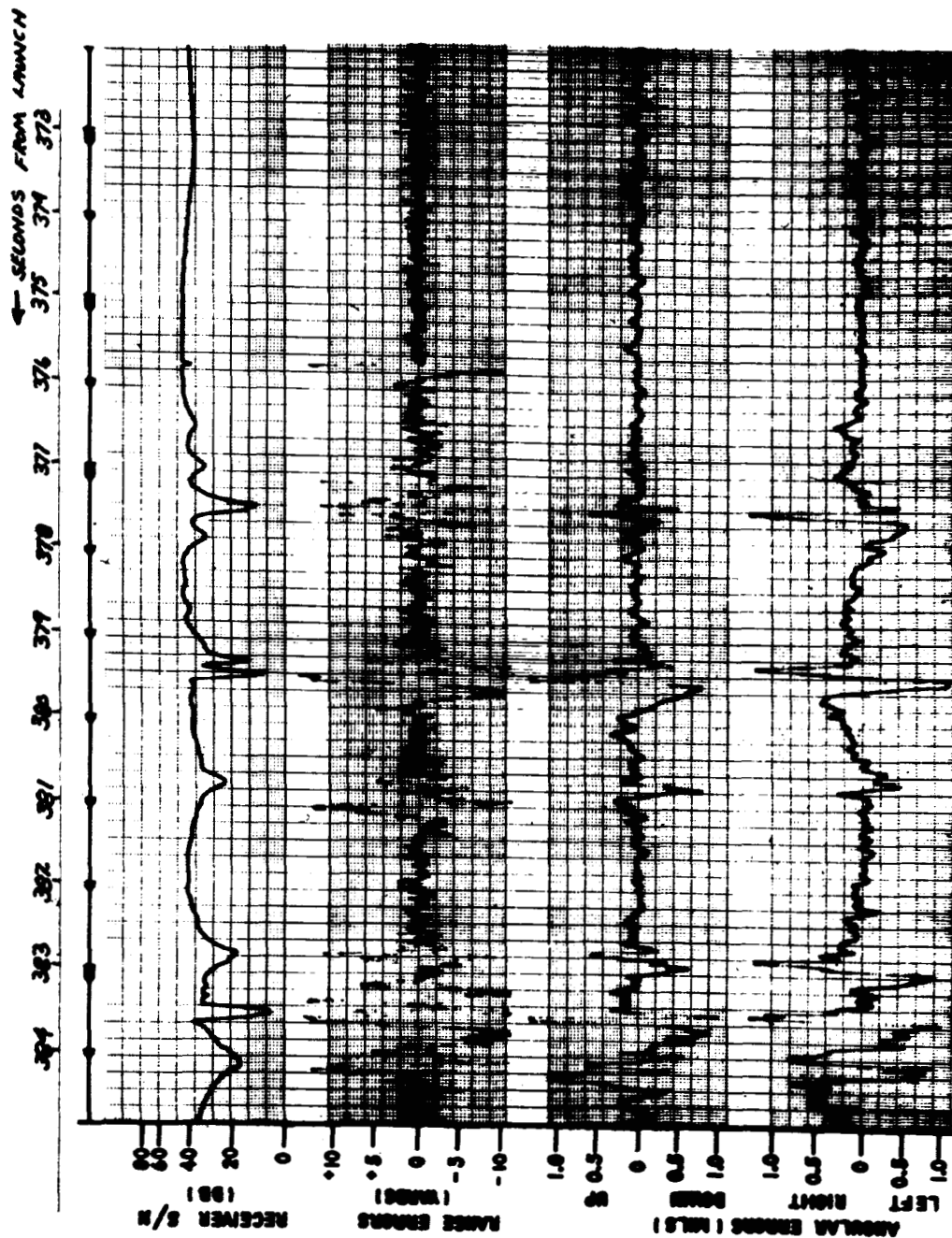


FIGURE 8



MR-1A Test of 19 December 1960  
 Function Record of 778-16 at Station 1.16  
 Beacon-transmit and beacon-receive lobing regions and resulting perturbations on range  
 and angular errors. (Note beacon-reply failures at 3-375.8, 377.5, 379.4, and 383.5 sec.)

C42-259A

(as recorded at the FPS-16) do not coincide again point out the frequency sensitivity of the capsule antenna system.\*

Figure 10 shows an interval of lobe-free tracking from the same test. Note the relatively low error voltages.

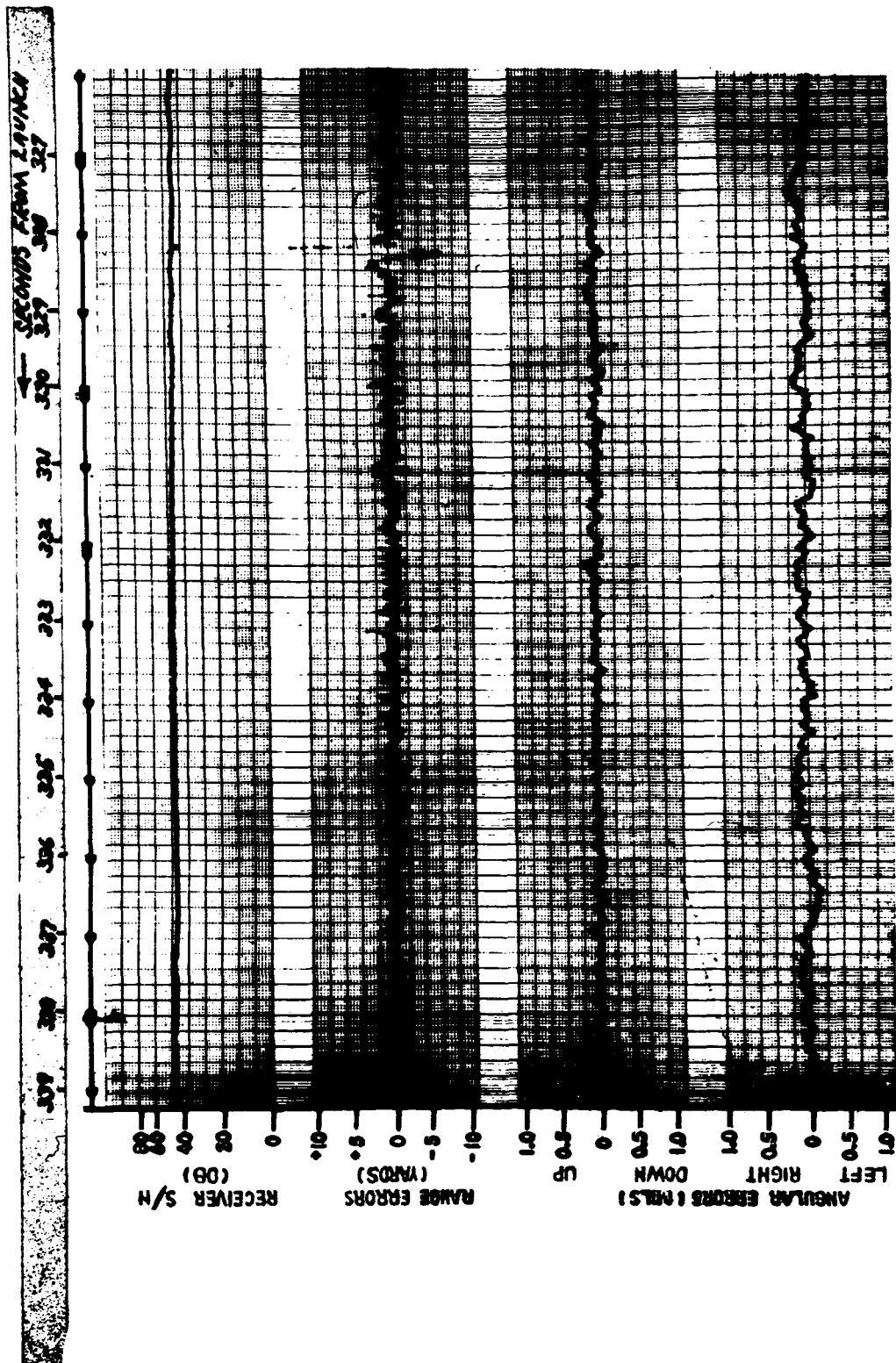
As a result of this test, it became clear that the abort decision at Bermuda was in jeopardy. It was feared that the end-for-end rotation of the capsule would cause a high amount of signal fluctuations due to antenna lobes. Thus, the test launch confirmed that the basic problem involved in the Mercury C-band tracking system was the capsule antenna pattern. The recommendations that followed this launch, in addition to recommendations on increasing power output and receiver sensitivities, also pointed out and emphasized the capsule antenna problem.

A relatively simple solution was put forward, namely, the insertion of time-modulated phase shifters in two of the lines to the three antenna elements of the capsule. This would produce a constantly changing interference region with the lobes and nulls fluctuating at a high enough rate so that the chances of the radar line of sight to the capsule being unfavorable (in a null) for an appreciable length of time would be very small.<sup>7</sup>

A study contract at this time was let to Melpar, Inc., designers of the three-element capsule antenna system.<sup>8,9</sup> The intent was to indicate the feasibility of such a phase modulation technique on the Mercury capsule. The phase shifters for this study were not designed to be flown. The phase shifters were built from strip-line with a motor-driven variable dielectric. Maximum shifts of 180° and 360° were used. Figure 11 is representative of the improvement possible with such a technique.

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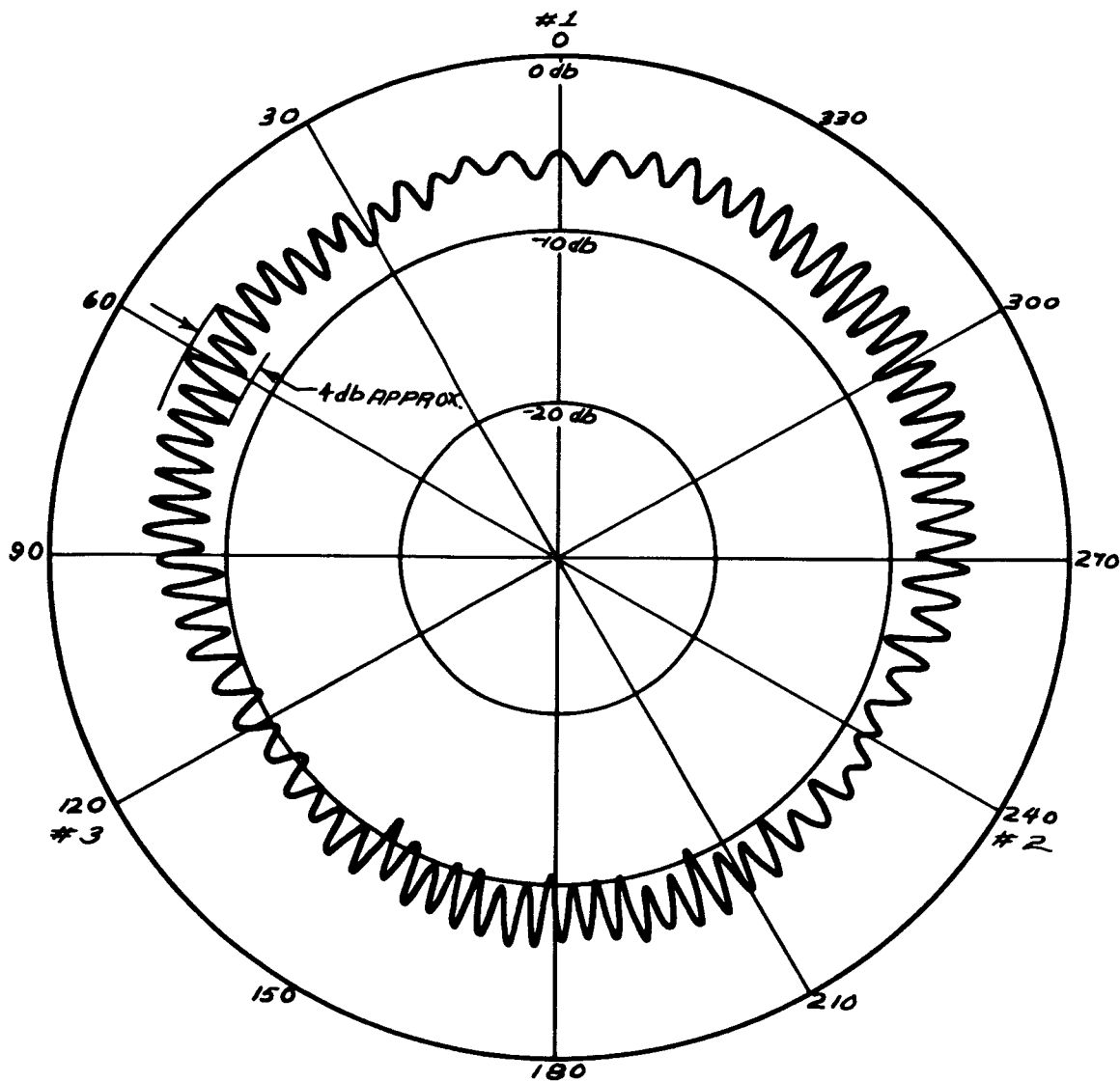
\*There is a difference of 75 Mcps between  $f_t$  and  $f_r$ .



MR-1A Test of 19 December 1960  
 Function Record of FFB-16 at Station 1.16  
 Interval of lobe-free tracking. (Note beacon failure to reply at 19028.2 sec.)

042-261A

FIGURE 10



**$\pm \pi/2$  PHASE SHIFTER IN ANTENNA LINES 243**

MELPAR INC. 5/29/61  
 $E \theta$ ,  $\phi$  VARIED,  $\theta = 90^\circ$   
 FREQUENCY  $F_T$  C-BAND

**C-BAND ANTENNA PATTERN**

FIGURE 11

C42-339

## VI. MATHEMATICAL INVESTIGATION OF PHASE SHIFTING\*

A theoretical analysis of the phase-shifting technique was not performed until after the first manned orbital flight.<sup>10</sup> However, the results were interesting in that it is theoretically possible to choose a maximum phase shift so that amplitude variations in antenna coverage as a function of viewing angle (in the interference regions) can be made to approach 0 db. The relationship for phase modulation as a function of time is

$$\theta(t) = \theta_{\max} \sin 2 \pi f t.$$

where

$$\theta_{\max} = \text{maximum phase shift (radians)}$$

and

$$f = \text{modulation frequency.}$$

The relationship for amplitude variation as a function of viewing angle in the interference regions is computed to be<sup>10</sup>

$$\frac{P_{\max}}{P_{\min}} = \frac{1 + J_0(\theta_{\max})}{1 - J_0(\theta_{\max})}$$

$J_0$  is the zero order Bessel function of the first kind, and  $\theta_{\max}$  is maximum phase shift.

Computations and experimental values are in relatively good agreement as shown in Fig. 11. For Fig. 11\*\* with a  $\pm \frac{\pi}{2}$  maximum phase shift,

$$\frac{P_{\max}}{P_{\min}} = \frac{1 + .45}{1 - .45} = 4.2 \text{ db.}$$

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\*A derivation of the equations given in this section is contained in the appendix.

\*\*Figure 11 is taken from Reference 8.

For Fig. 12<sup>\*</sup> with a  $\pm 0$  maximum phase shift,

$$\frac{P_{\max}}{P_{\min}} = \frac{1 + (-.3)}{1 - (-.3)} = -2.6 \text{ db.}$$

Amplitude variations are reduced to 0 db as a function of viewing angle when

$$\frac{P_{\max}}{P_{\min}} = 1.$$

This condition obtains when  $\theta_{\max} = 2.4, 5.5, 8.6, 11.7$ , etc. radians. One would most likely use  $\theta_{\max} = 2.4$  radians in a practical system.

#### VII. MA-4

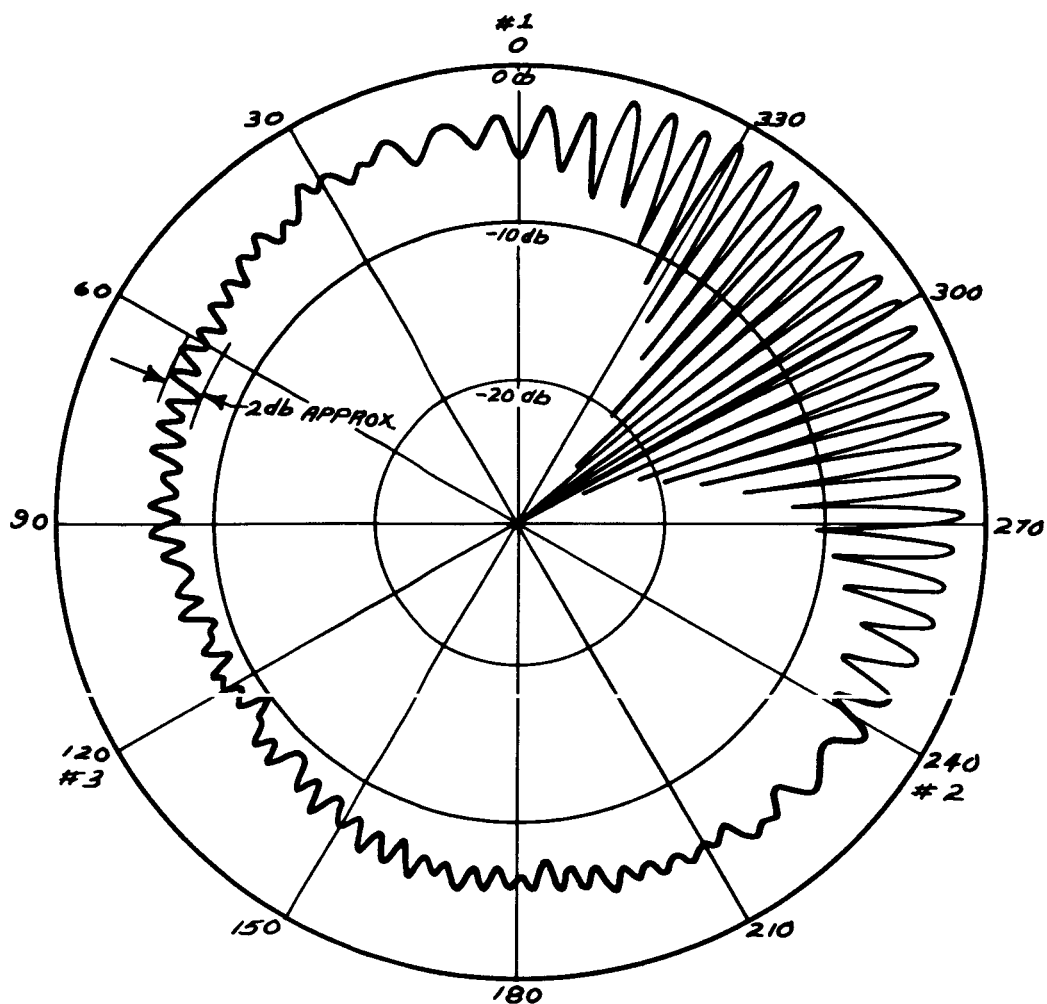
The first successful orbiting of a Mercury capsule (by an Atlas D missile) occurred 13 September 1961.<sup>11</sup> This flight was the first test of the world-wide Mercury tracking range and took place with an unmodified capsule antenna system. (The phase shifters were being readied for the MA-5 launch.)

Figure 13 describes the tracking from Cape Canaveral during the launch phase of flight. The drop-out from  $T = +60$  to  $T = +80$  seconds is believed due to flame attenuation.

Figure 14 describes the predicted  $S/N$  and  $S/S_T$  ratios at Bermuda. Of special importance is the dip in  $S/S_T$  ratios at  $T = 310$  to  $T = 360$  seconds. These predictions are based on average values of capsule antenna gain. Therefore, this interval is considered as being a poor tracking region since antenna nulls may be in the order of 25-30 db. Thus, the 10-db margin illustrated in Fig. 14 during this interval may easily be expended in antenna nulls. There is no correlation to be made with actual data, however, since the FPS-16 at Bermuda neither acquired nor tracked.

Figures 15 and 16 describe the RMS angular noise averaged over two seconds at the Cape Canaveral FPS-16. Cut-off of the sustainer engine and capsule separation occurred at approximately  $T = 300$  seconds.

\*Figure 12 is taken from Reference 9.



**$\pm \pi$  RADIANS PHASE SHIFT IN LINE NO.3**

MELPAR INC. 9/11/61  
 $E\phi, \phi$  VARIED,  $\theta = 90^\circ$   
 FREQUENCY  $F_T$  C-BAND

**C-BAND ANTENNA PATTERN**

FIGURE 12

C42-340

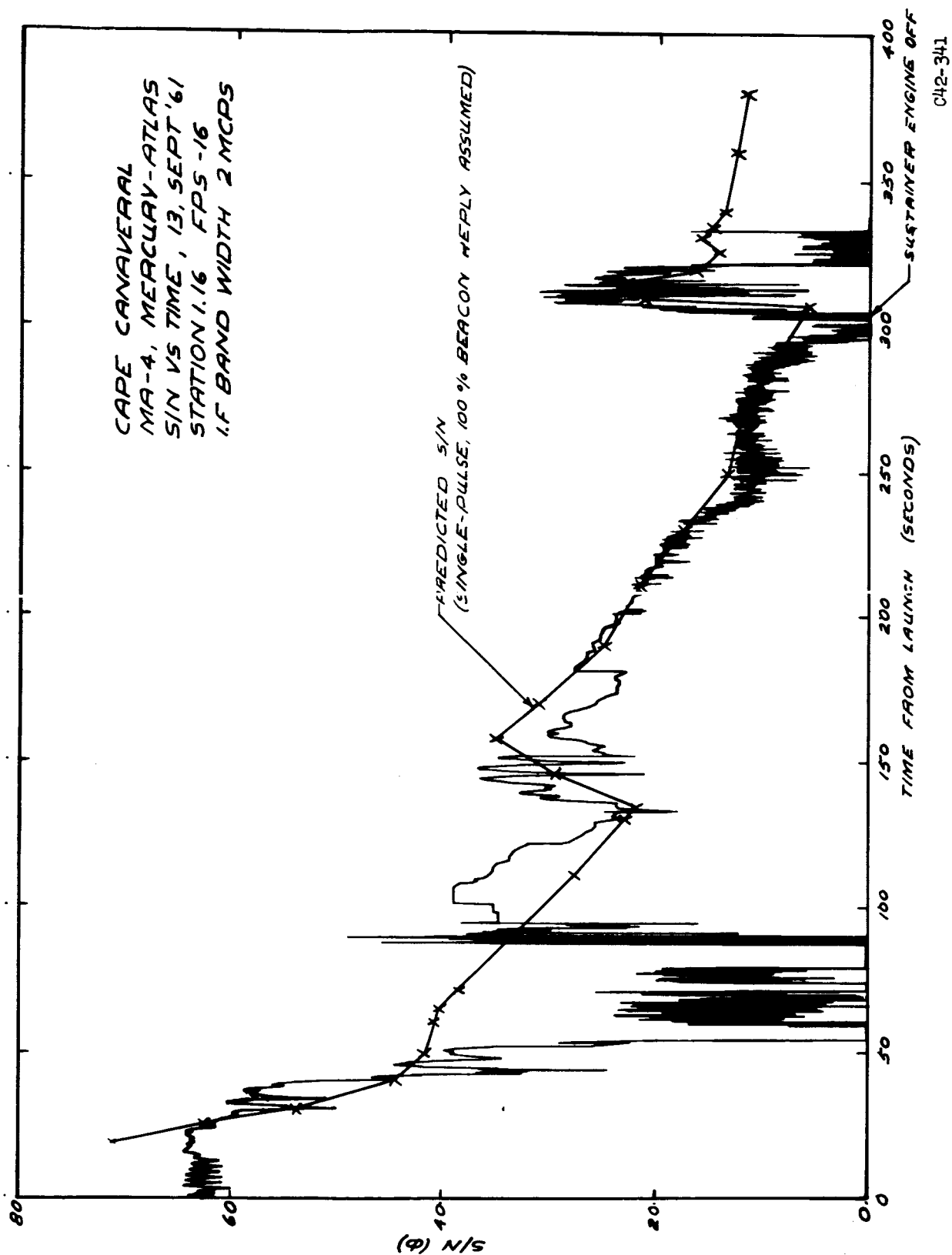
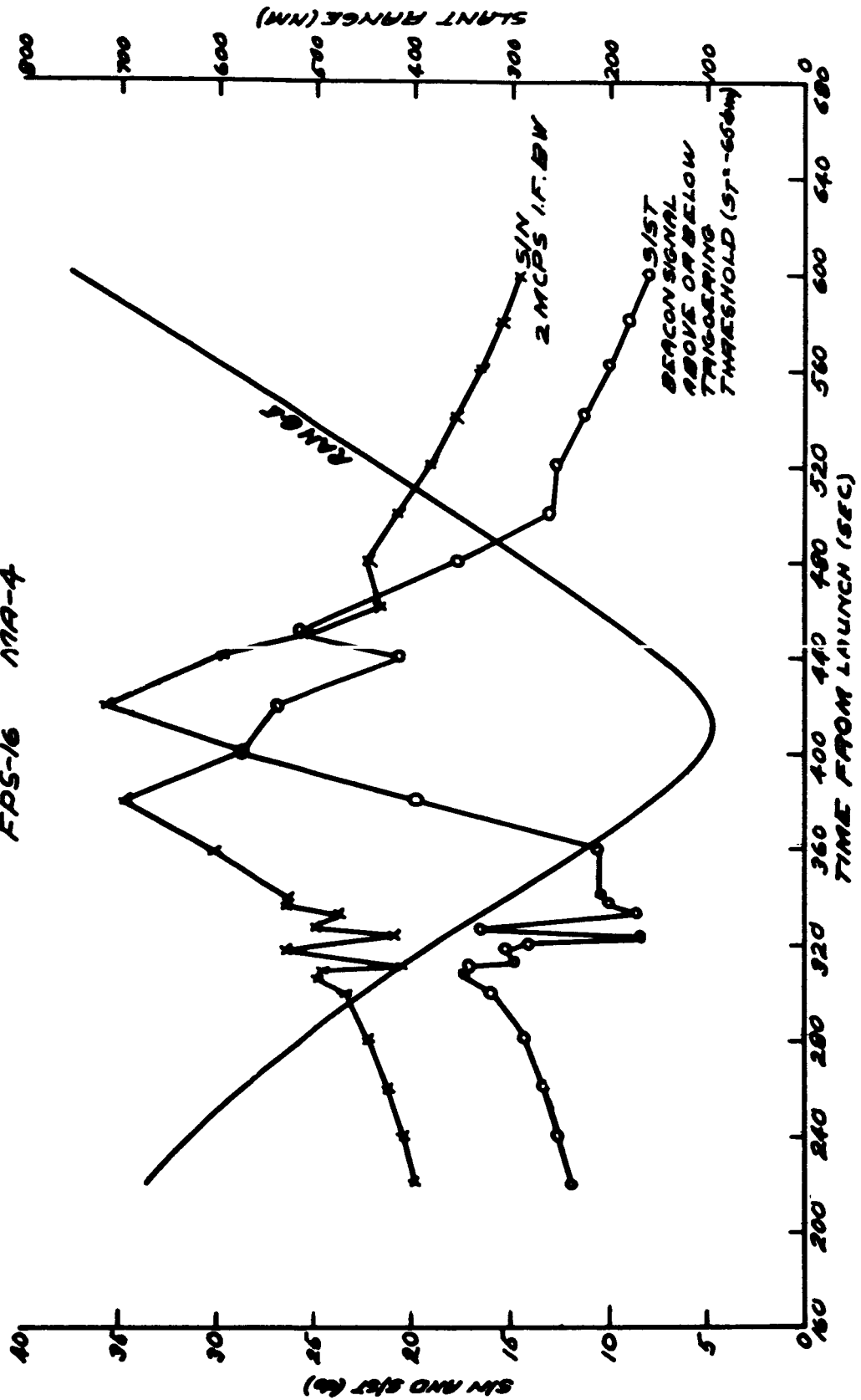


Figure 13.

# BERMUDA, PREDICTED SN V6 TIME

FPS-16 NNA-4



042-342

FIGURE 14

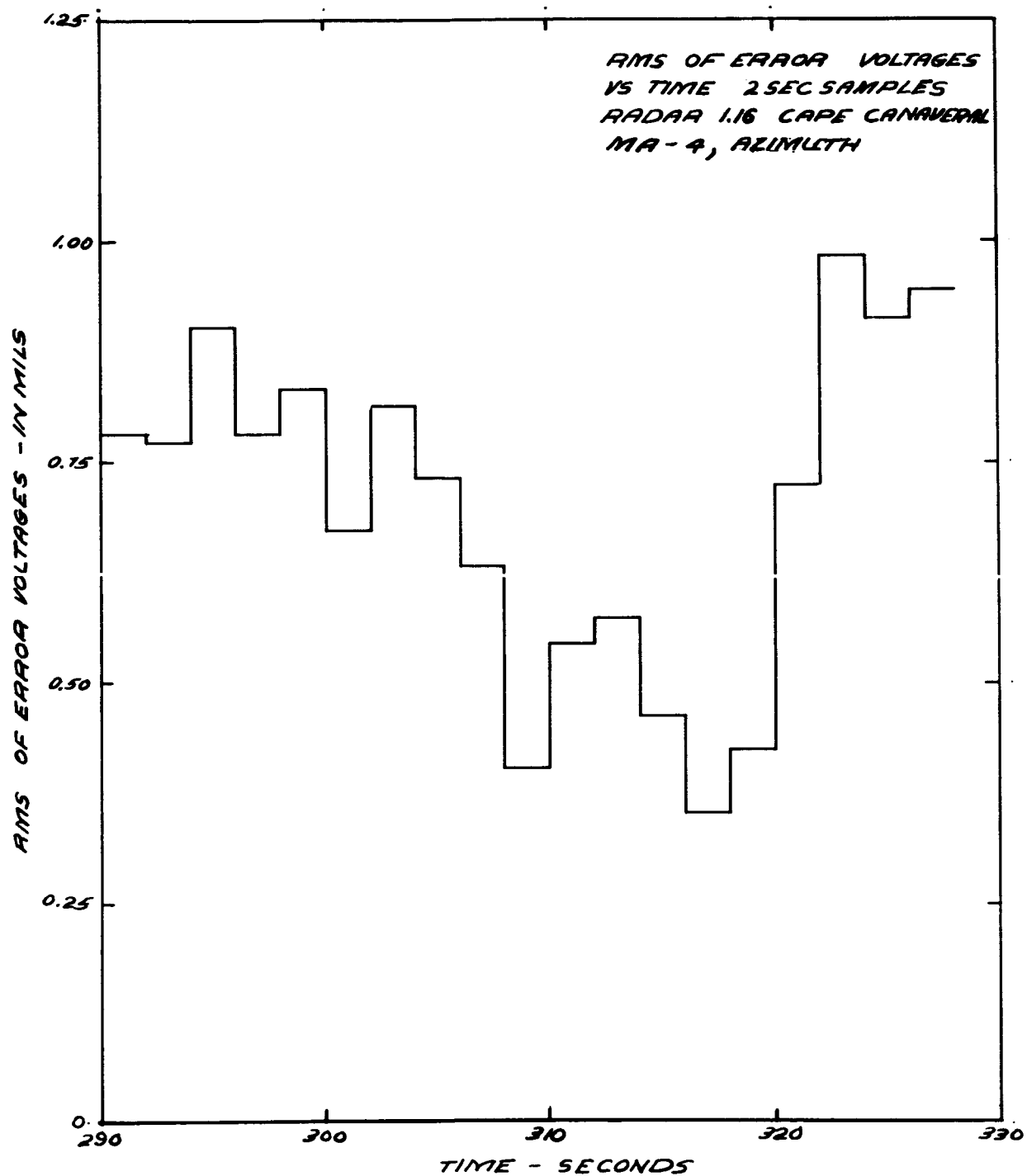
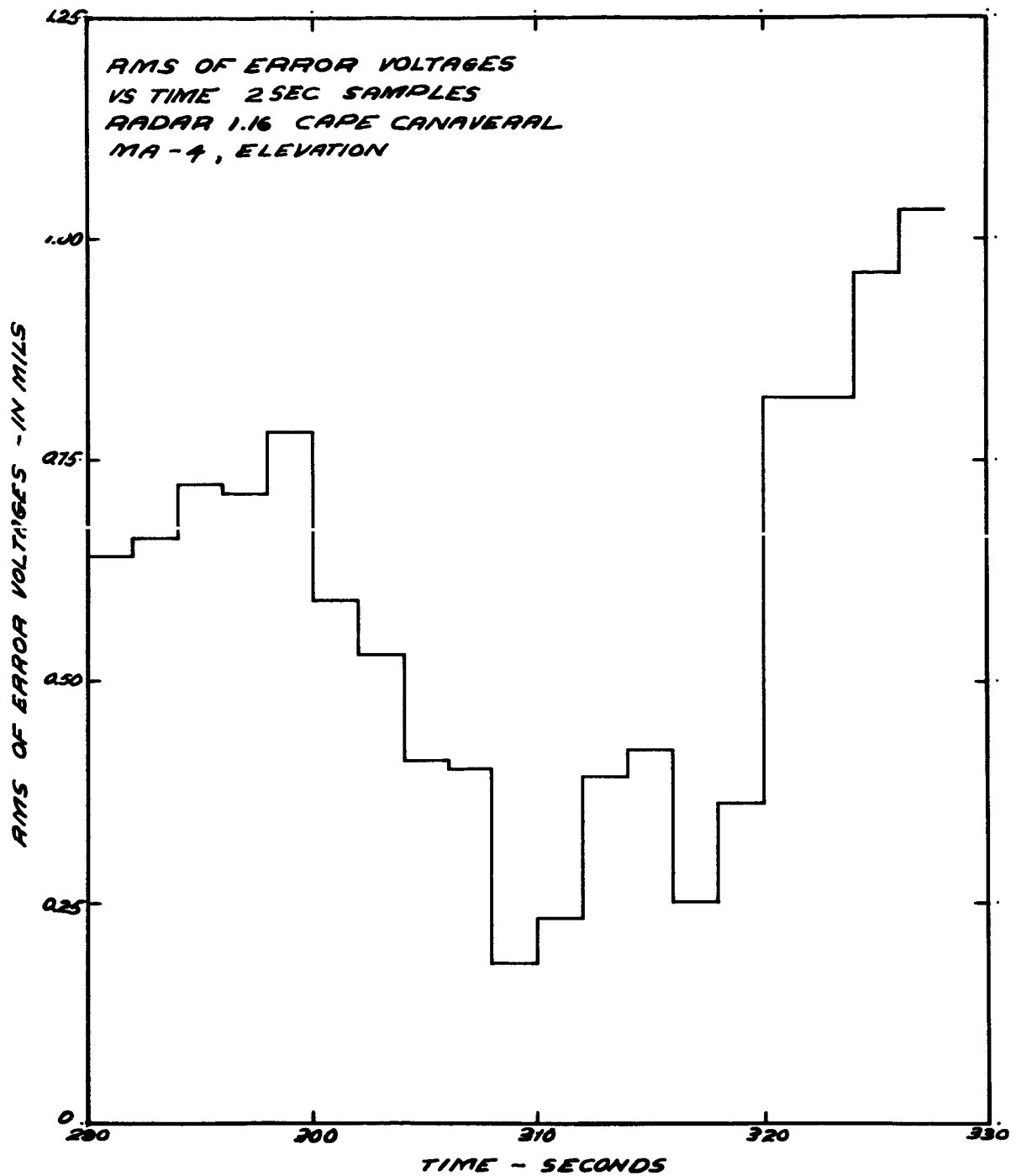


FIGURE 15

C42-343



C42-364

FIGURE 16

The shape of these histograms are generally a simple inverse function of S/N ratios at the FPS-16 receiver. The signal level came up as the capsule began to turn at  $T = 305$  seconds, causing a decrease in angular noise. The magnitude of the angular noise is very discouraging. It is doubted whether an orbit determination calculation could have been made on data of this quality. (Indeed, the adequacy of the orbit was very much in doubt at the time.)

It was obvious that the tracking was unsatisfactory.

#### VIII. MA-5

The Mercury-Atlas launch (MA-5) of November 29, 1961, was a three-orbit mission with a chimpanzee aboard. It was the first attempt to utilize a ferrite phase shifter in orbit. \* The difference in performance of the C-band radars was easily discernible. Almost all stations tracked. The FPS-16 radar at Bermuda produced valid data of adequate quality to confirm the orbit quickly. It was not possible to make predictions of the S/N ratios possible with the phase shifter in operation since antenna patterns with the phase shifter were not made. (Only one phase shifter was used to enhance those portions of the antenna patterns as seen by the radar at Bermuda during turn-around.) It is possible, however, to make comparisons of S/N at some sites which tracked an "unimproved" capsule and one with the phase shifter aboard.

Figure 17 describes the S/N ratios obtained at the FPS-16 at Cape Canaveral. The most noticeable areas of improvement occur between  $T = 100$  and  $T = 300$  seconds since the radar line of sight to the capsule over this interval fell between the two radiating elements, one of which is being phase modulated (see Fig. 13 for comparison).

Figure 18 describes the S/N ratios obtained at the Bermuda FPS-16. No comparison to the unmodulated condition can be made since no data were obtained

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\*Mechanical phase shifters for the purpose of enhancing antenna coverage were used a number of years ago in another project.

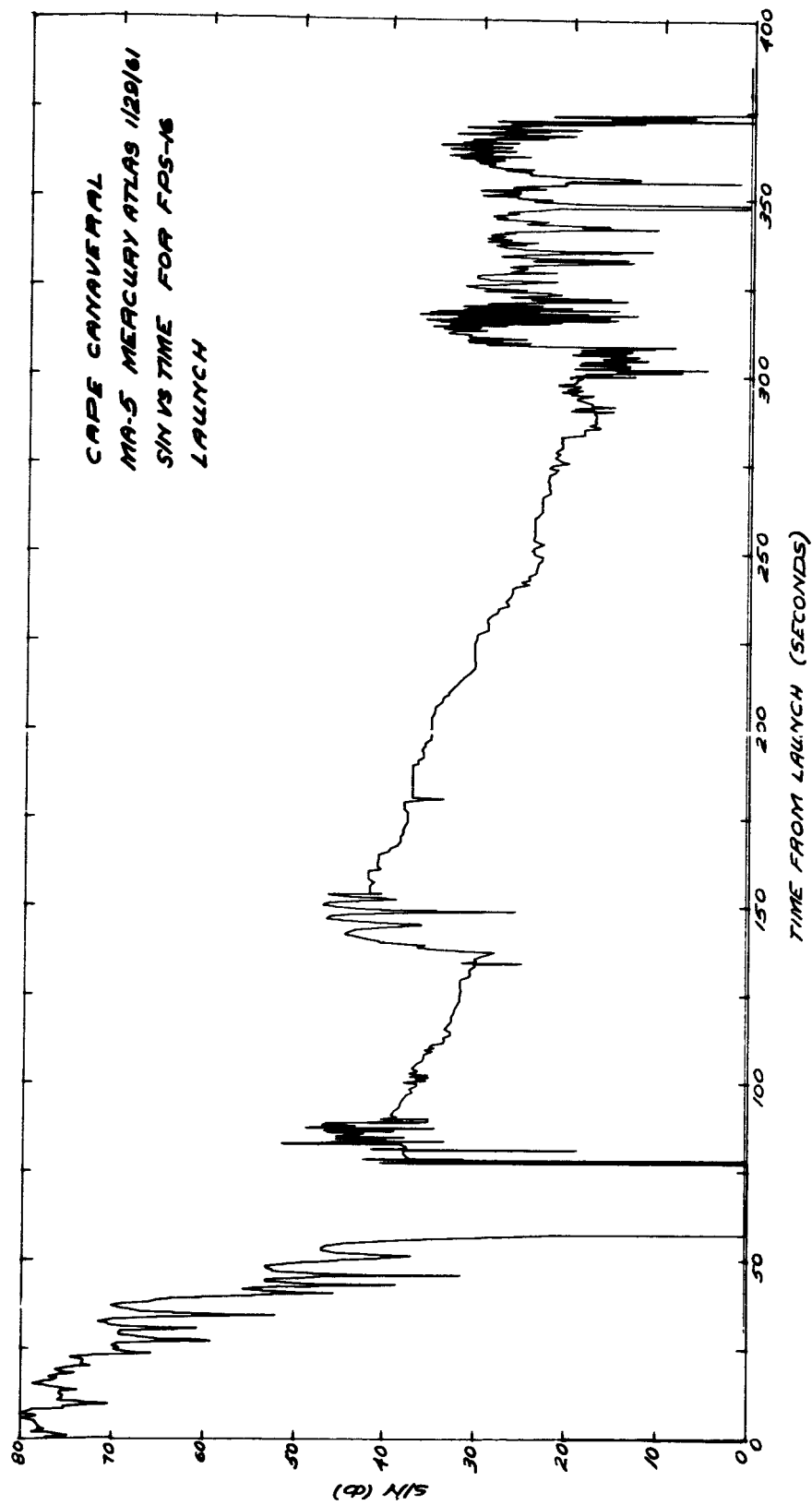


FIGURE 17

C42-365

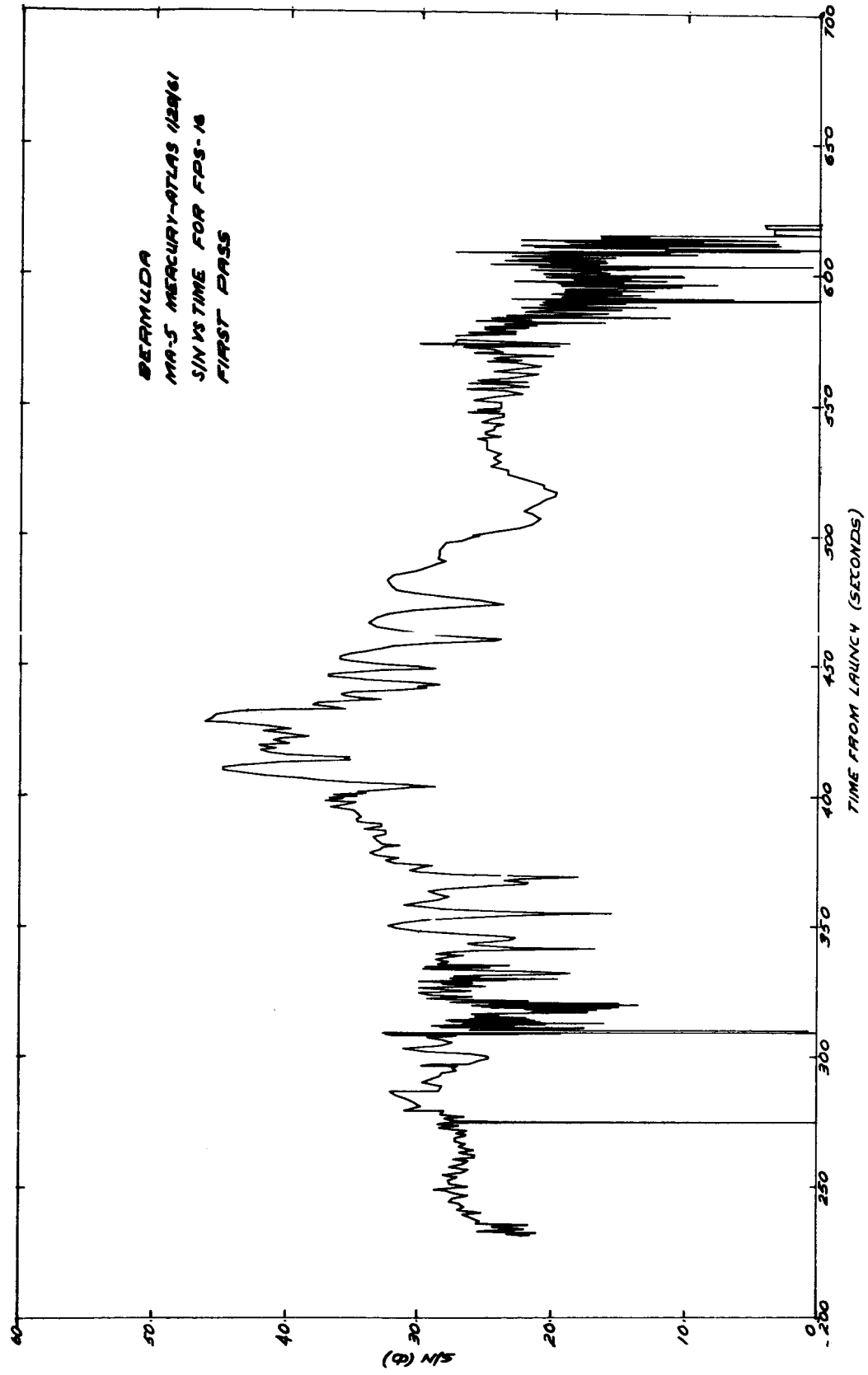


FIGURE 11

C42-366

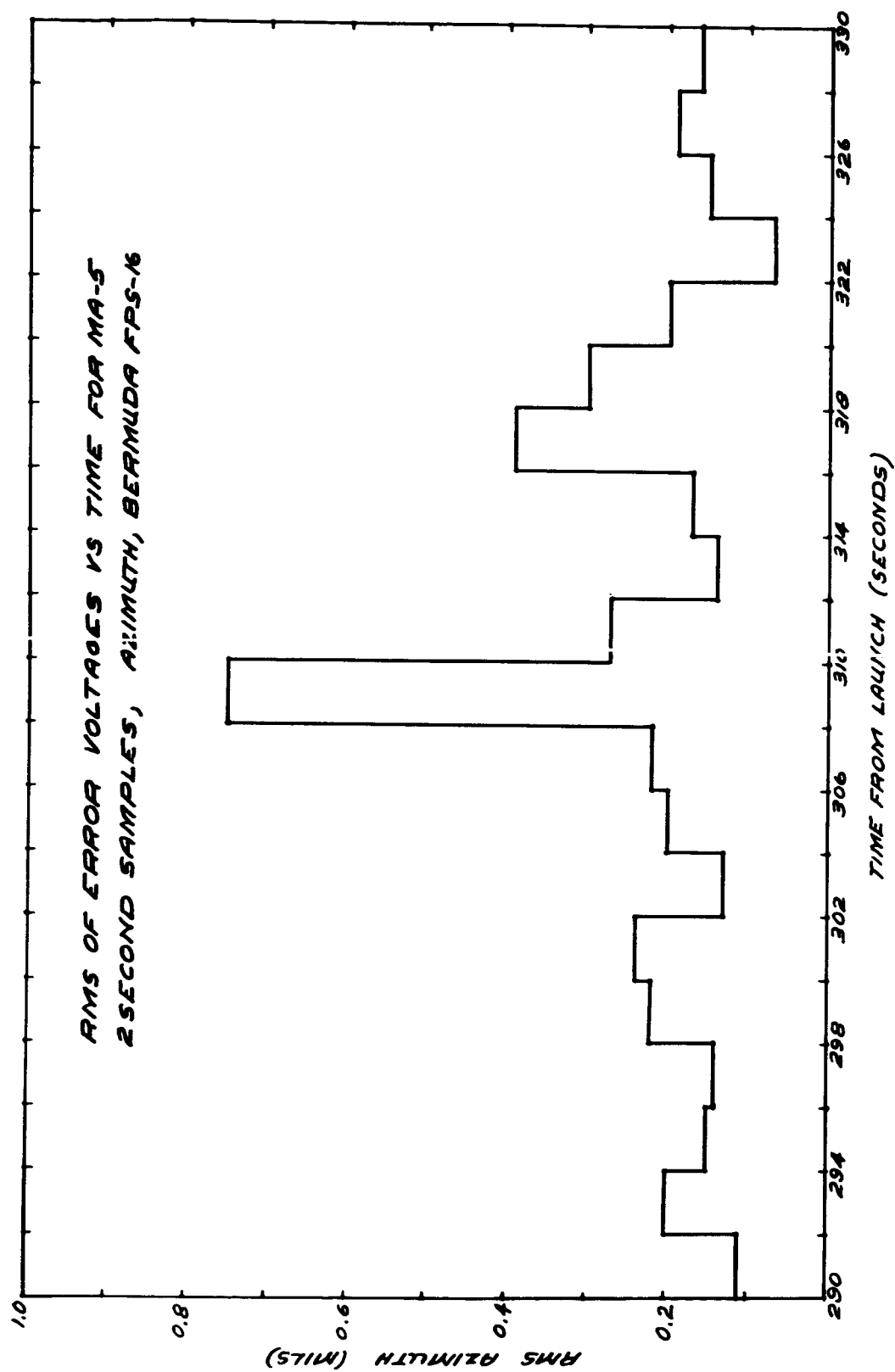
from the MA-4 launch. Figures 19 and 20 are histograms of angular errors obtained at Bermuda for azimuth and elevation angles, respectively. It should be noted that the azimuth noise voltages are considerably lower than the elevation error voltages. One would ordinarily expect the azimuth voltages to be the greater since the capsule is being rotated 180° end for end during this interval. Examination of the data during this interval seems to indicate a broader servo bandwidth for the azimuth servo loop than for the elevation servo loop. However, at turn-around ( $T = 300$  seconds) the elevation of the radar is changing at about  $0.5^\circ/\text{sec}$ , while the azimuth angle is changing at about  $.25^\circ/\text{sec}$ . It is possible that the elevation servo bandwidth was set at too narrow a value.

#### IX. MA-6 (MANNED FLIGHT)

The first U. S. manned orbital space flight was made by Lt. Colonel John Glenn on February 20, 1962. The capsule C-band antenna system had a phase shifter aboard as did the earlier MA-5 flight. The performance, generally speaking, was similar (as far as radar tracking was concerned) to the performance obtained with the MA-5 test.

Figure 21 describes the signal-to-noise ratios obtained at Patrick Air Force Base, Florida. Figure 21 may be generally compared with Fig. 13 for a comparison of tracking performance of a similar trajectory with and without the phase shifter, respectively. Figure 21 (with phase shifter) shows a generally smoother plot with less fluctuations.

Figure 22 describes the signal-to-noise ratios obtained at Bermuda. The generally high signal-to-noise ratios at time of capsule separation ( $T = 300$  seconds, approximately) afforded adequate signal strength so that the RMS angular errors were relatively small.



C42-367

FIGURE 19

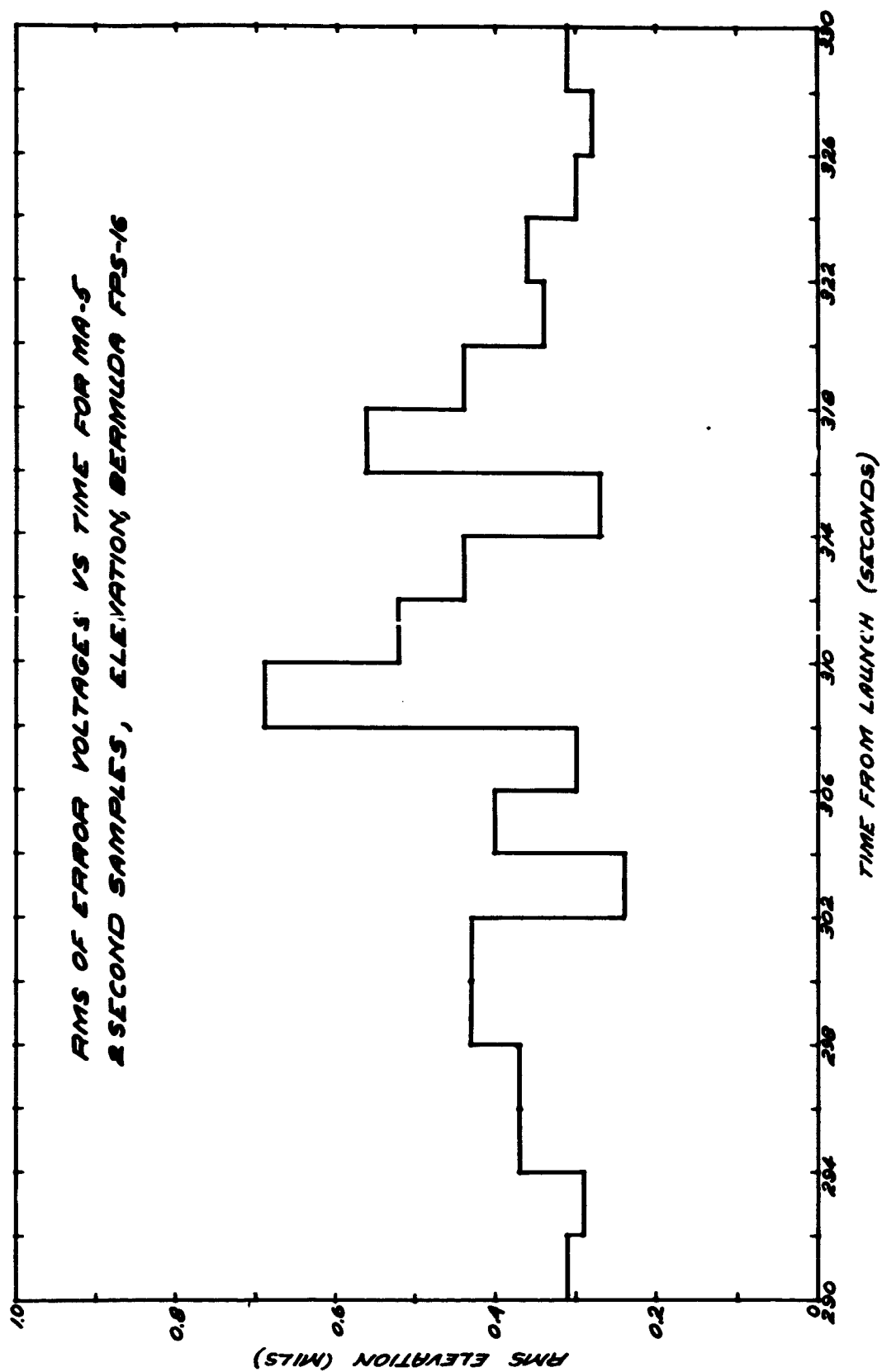
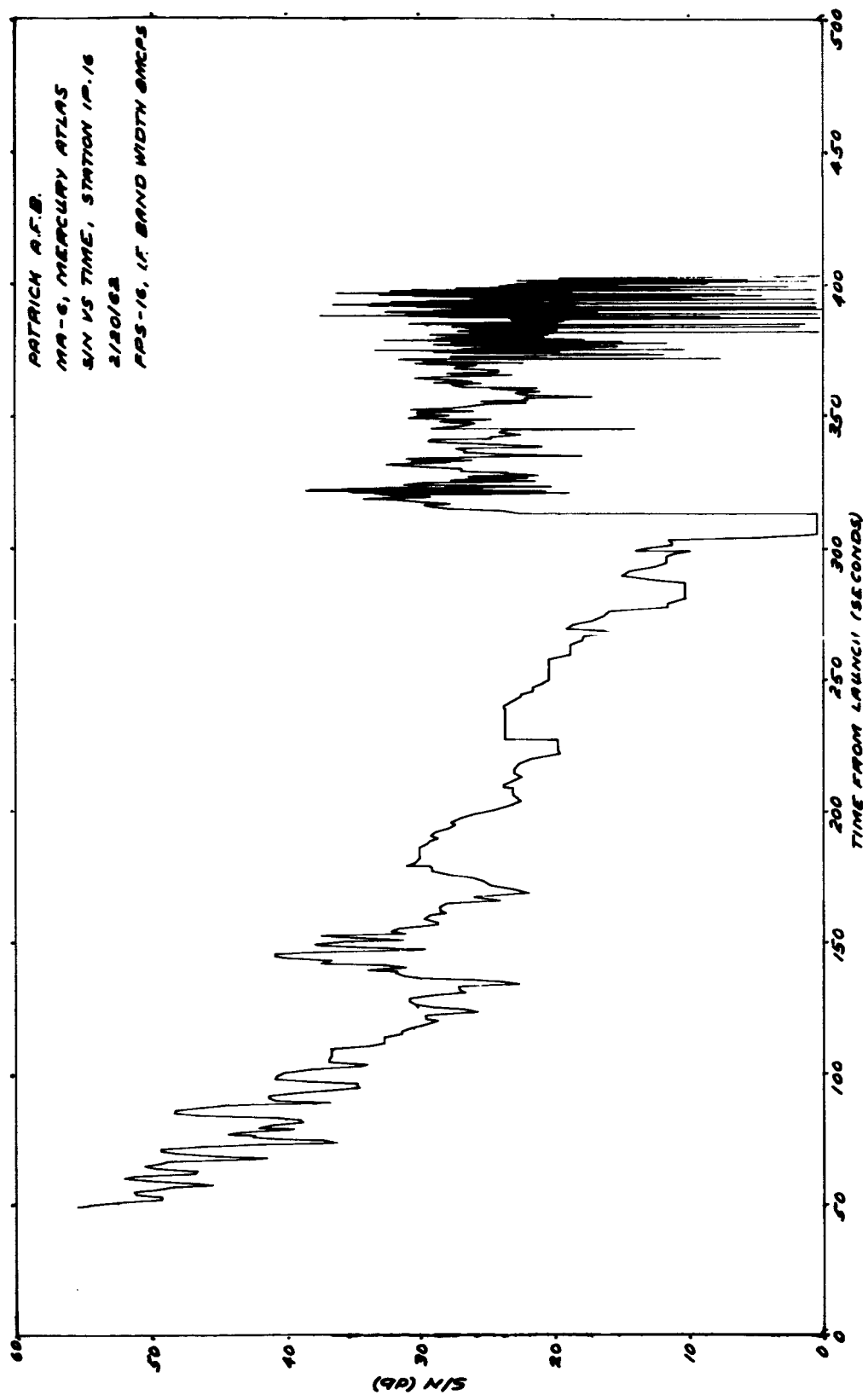


FIGURE 20

C42-368



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FIGURE 21.

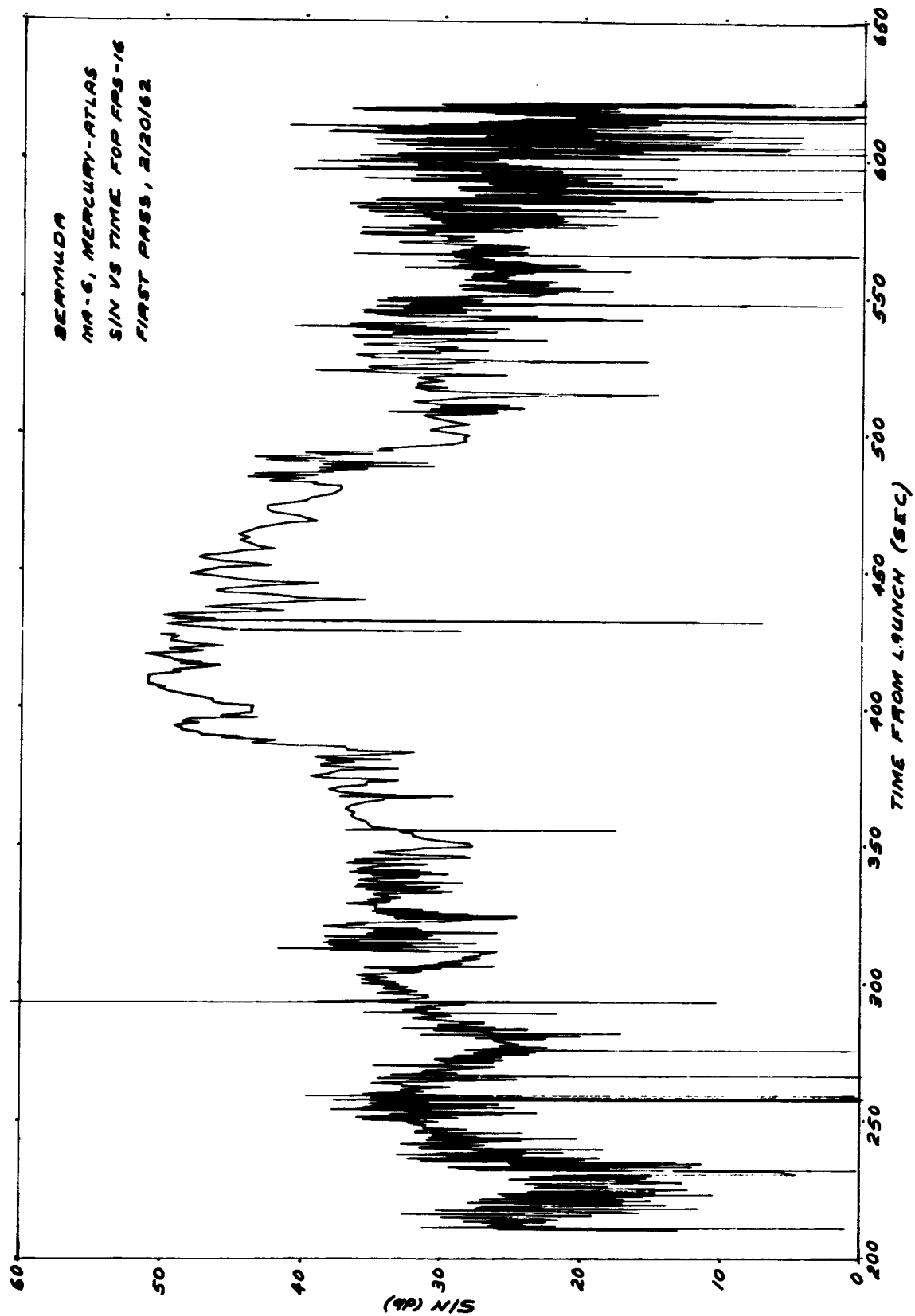


FIGURE 22

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Figures 23 and 24 describe the RMS angular errors for azimuth and elevation angles, respectively. It can be seen that the values generally average 0.25 to 0.30 mils. The quality of tracking was adequate for verification of the orbit.

#### X. CONCLUDING STATEMENTS

This paper has attempted to describe the background, evolution, and treatment of some of the problems associated with the C-band beacon tracking for Project Mercury. The fact that the radar-beacon tracking ultimately was adequate to the task is a testimonial to the large amount of effort expended by the many people associated with the Project. The development of the radar-beacon tracking problems and subsequent treatment as described in this paper point out the necessity of an over-all, integrated approach to the tracking of space vehicles.<sup>12</sup> With such an approach, tracking, telemetry, and communications for future space probes can be combined so that maximum reliability and functional usefulness are obtained with a minimum of equipment redundancy.

BHL/jm

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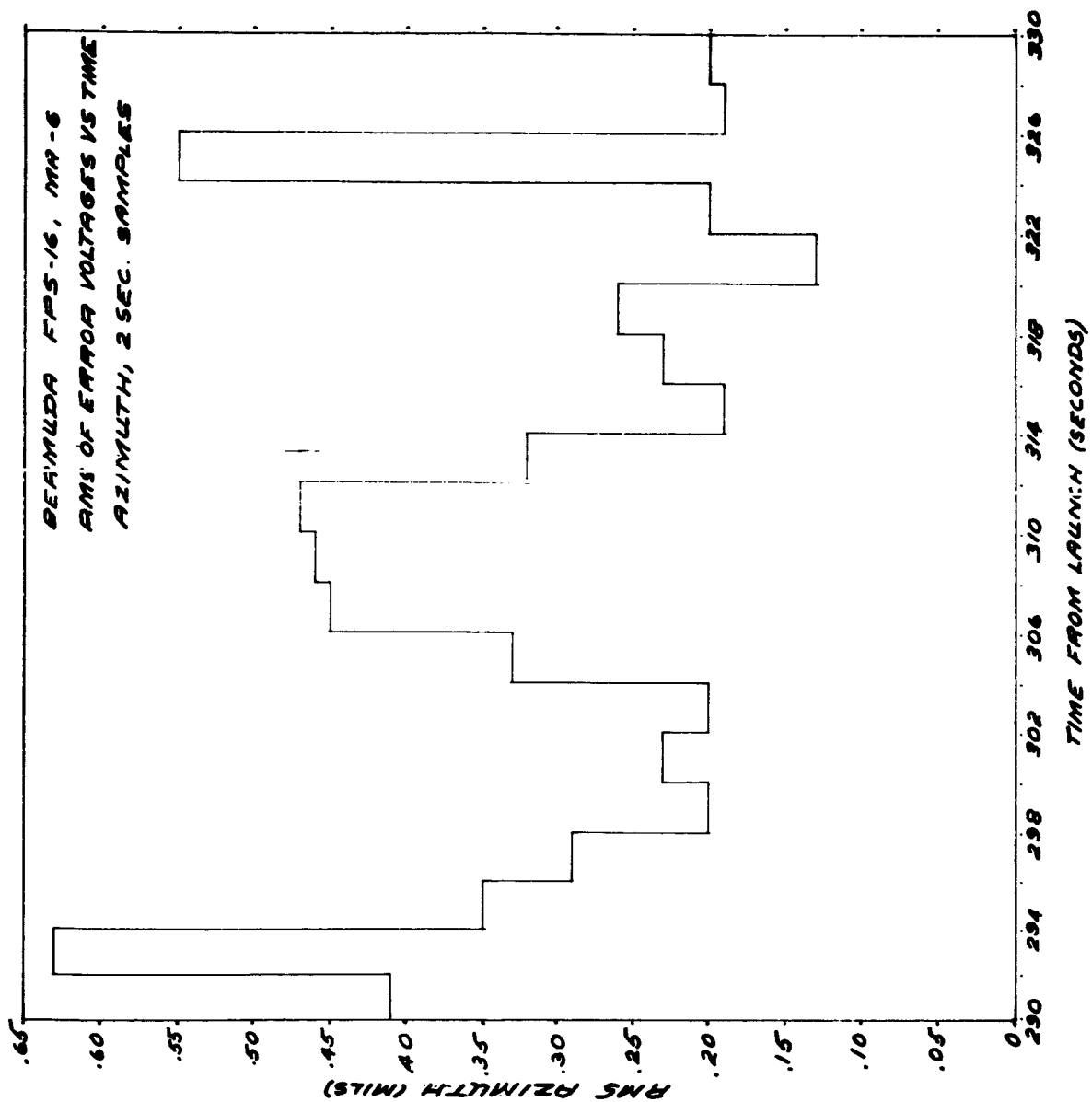
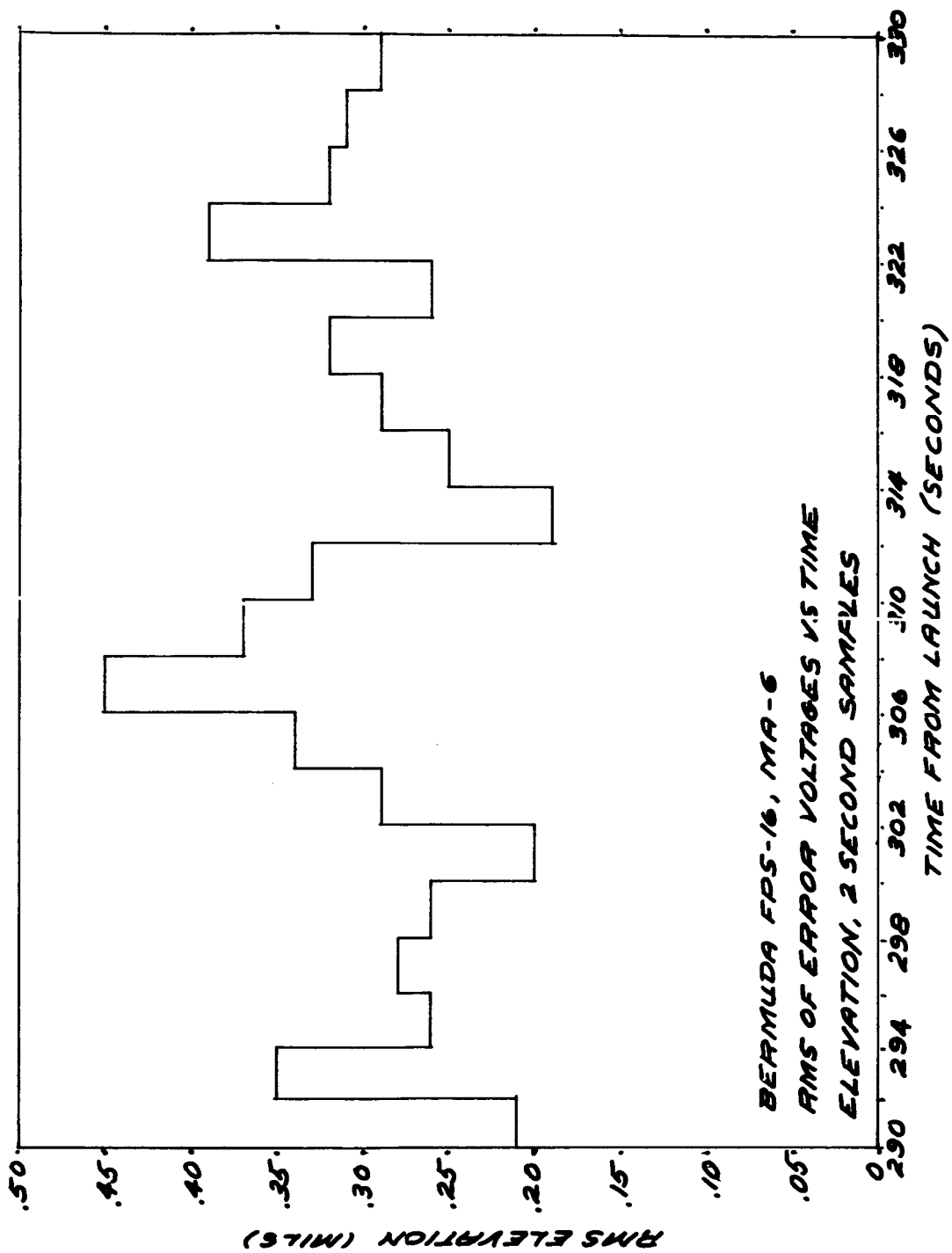


FIGURE 2}

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## APPENDIX

### AN ANALYSIS OF THE PHASE-SHIFTING TECHNIQUE\*

#### INTRODUCTION

The model chosen for analysis herein considers two adjacent elements only. However, the results of this analysis are generally applicable to the three-element Mercury beacon-antenna system since there is minimum coupling between non-adjacent elements.

#### GENERAL CONFIGURATION

The model chosen for this analysis is shown in Fig. 25. If  $D_1$  and  $D_2$  are equal, the summation of energy at point O may be considered to be (by superposition) the summation of RF voltages from Radiator #1 and Radiator #2.

For a static situation (no phase shift vs. time) we may consider

$$e_1(t) = E \cos (wt + \varphi) \quad (1)$$

and

$$e_2(t) = E \cos (wt + \alpha) \quad (2)$$

where  $\varphi$  and  $\alpha$  are arbitrary but fixed phase shifts.

The summation of  $e_1(t)$  and  $e_2(t)$  at point O is

$$e_1(t) + e_2(t) = E (\cos [wt + \varphi] + \cos [wt + \alpha]) \quad (3)$$

and

$$e_1(t) + e_2(t) = 2 E \cos \frac{1}{2} (\varphi - \alpha) \cos (wt + \frac{\varphi + \alpha}{2}). \quad (4)$$

---

\*Excerpted from Reference No. 10.

Equation (4) indicates a number of interesting features:

- a) The maximum amplitude of  $e_1(t) + e_2(t)$  is  $2E$ .
- b)  $\frac{\varphi + \alpha}{2}$  is the phase shift of the resultant sinusoid.
- c)  $2E \cos \frac{1}{2} (\varphi - \alpha)$  is the peak amplitude of the resultant sinusoid (which is time invariant for a fixed  $\varphi$  and  $\alpha$ ).

We may now introduce a time-modulated phase shift at one of the radiators so that

$$\varphi(t) = \theta_m \sin w't \quad (5)$$

where  $w'$  is the modulation frequency of phase shifting (for a maximum phase shift of  $\theta_m$  radians).

Substitution of (5) into (4) yields

$$e_1(t) + e_2(t) = 2E \cos \frac{1}{2} (\theta_m \sin w't - \alpha) \cos \left( wt + \frac{\theta_m \sin w't + \varphi}{2} \right) \quad (6)$$

In Equation (6),  $(\theta_m \sin w't + \varphi)/2$  is the phase-modulation term, and  $2E \cos 1/2 (\theta_m \sin w't - \alpha)$  is the amplitude coefficient.

In the Mercury environment  $w' = 2 \pi 400 t$  for a 400-cps modulation rate. For most applications the phase-modulation rate will be in the order of 100 to 500 cps. The only requirement is that modulation rate be greater than the maximum radar servo bandwidth. One should also be cautioned on excessive modulation rates since a very high modulation rate may deteriorate monopulse operation in addition to creating excessive sidebands.

#### AVERAGE POWER CONSIDERATIONS

If one wishes to employ an optimum phase shift so that maximum benefit may be derived, one should be able to calculate the power loss or gain for arbitrary relative phase angles ( $0 < \alpha < \pi$ ) as a function of maximum phase shift  $\theta_m$ .

Thus, one would like to know the average square of the amplitude coefficient, which we may define as  $\overline{A^2}$ .

$$\overline{A^2} = \frac{2E^2}{\pi} \int_0^{2\pi} \cos^2 \left( \frac{\theta_m}{2} \sin w't - \frac{\alpha}{2} \right) dw't. \quad (7)$$

(The square root of Equation (7)  $\left( \sqrt{\overline{A^2}} \right)$  would simply be the rms of the amplitude coefficient.)

Using the  $\cos^2$  identity,

$$\overline{A^2} = \frac{E^2}{\pi} \int_0^{2\pi} (1 + \cos [\theta_m \sin w't - \alpha]) dw't \quad (8)$$

$$\overline{A^2} = \frac{E^2}{\pi} \left[ \int_0^{2\pi} dw't + \int_0^{2\pi} \cos [\theta_m \sin w't - \alpha] dw't \right] \quad (9)$$

$$\overline{A^2} = \frac{E^2}{\pi} \left[ 2\pi + 2\pi \cos \alpha J_0(\theta_m) \right] \quad (10)$$

and

$$\overline{A^2} = 2E^2 (1 + \cos \alpha J_0(\theta_m)) \quad (11)$$

When  $\alpha = 0, \pi$  (as in a lobe and null, respectively),  $\overline{A^2}$  as a function of maximum phase shift can be described as in Fig. 26.

It can be seen from Fig. 26 that if one were to choose a value of  $\theta_m$  that would maximize the gain in a null region, a loss of power would be suffered in the lobe regions since the curves shown in Fig. 26 are out of phase. However, if one were to choose  $\theta_m$  so that  $\overline{A^2}(\alpha = 0)/\overline{A^2}(\alpha = \pi) = 1$ , one would have  $\theta_m = 2.4, 5.5, 8.6, 11.7$  radians. For those values of  $\theta$ , therefore, there should be no "ripple" in signal gain as a function of  $\alpha$  (or small changes in

---

\*"Ripple" is defined as the ratio of  $P_{\max}/P_{\min}$  in the interference regions as a function of  $\alpha$  (or viewing angle).

viewing angle to the antenna system).

For other values of  $\theta_m$ , one can compute the "ripple" in antenna coverage from

$$\frac{\overline{A^2}(\alpha = 0)}{\overline{A^2}(\alpha = \pi)} = \frac{1 + J_0(\theta_m)}{1 - J_0(\theta_m)} \quad (12)$$

#### EXPERIMENTAL RESULTS

The experimental results were discussed earlier in this report (pp. 21, 22) for values of  $\theta_{\max} = \pi/2$  and  $\pi$ .

#### CONCLUSIONS

If the phase-modulation technique is to be utilized, an optimum  $\theta_m$  exists at 2.4 radians (approximately  $\pm 137^\circ$ ), whereby a minimum of "ripple" in antenna coverage can be afforded. Other values of phase-modulation technique will have "ripple" according to Equation (12).

## GENERAL CONFIGURATION

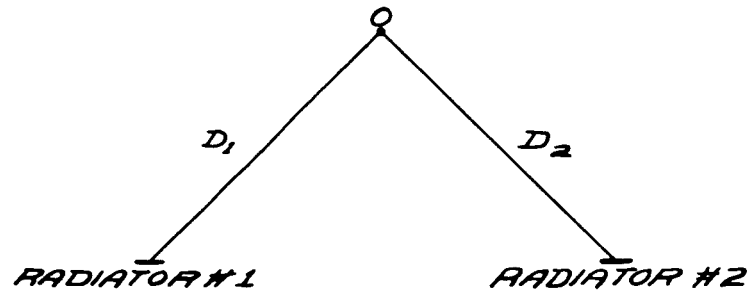


FIGURE 25

## POWER VS MAXIMUM PHASE SHIFT

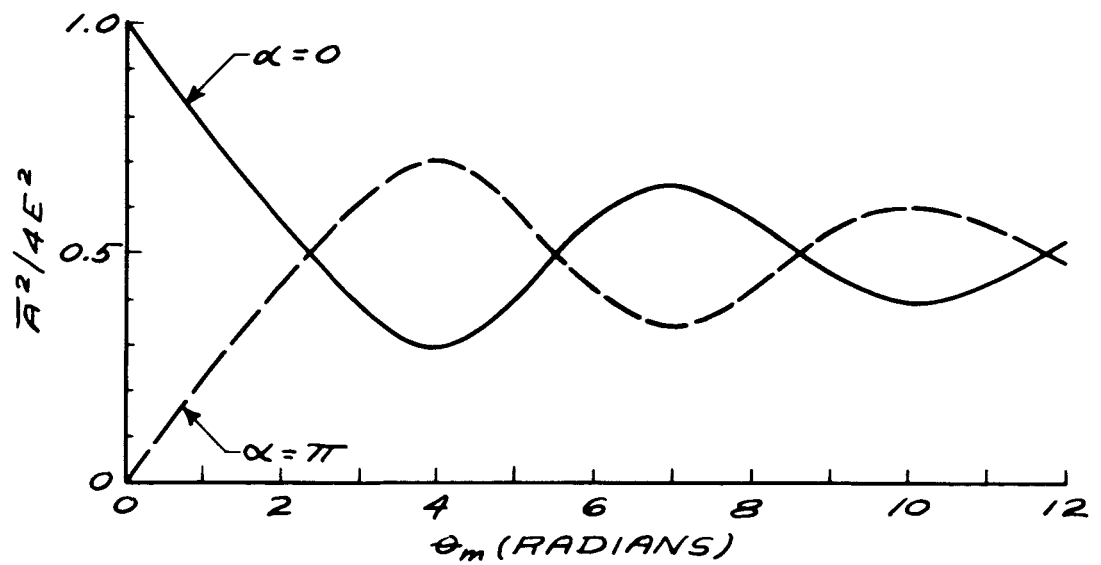


FIGURE 26

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